

# A New Algorithm for High-frequency Capacitance Performance Parameter Testing Based on Multiple Ways of Errors Eliminate



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**Abstract.** High-frequency capacitors of high-quality are widely used in military and high-end consumer electronic products, it's particularly important to measure Q value and other performance parameters of high-frequency capacitance. The fixture effects error of measurement system is particularly acute under the influence of capacitance for high frequency band. Aiming at this issue, a way to eliminate errors of the testing system based on modeling of telomeric extension and auto telomeric extension technology combines with de-embedding technologies is proposed to eliminate errors produced by fixture influence, and then using polynomial fitting algorithm that based on least square method to compute performance parameters and equivalent circuit parameters accurately. Experimental results indicate that the test data of proposed algorithm are more accurate compared with that of using the de-embedding technique simply for measuring capacitance at 10GHz or higher frequencies.

**Keywords:** capacitance parameter, de-embedding, high-frequency capacitors, polynomial fitting, vector network analyzers

## 1 Introduction

Modern electronic circuit has the characteristics of integration and miniaturization. Chip capacitor has become the mainstream of electronic design, and is widely used in the field of RF circuit design [1-2]. Therefore, it is important to measure the performance parameters of the chip capacitor in the RF frequency band, and is important for the design of electronic circuits.

At present, the measurement of the patch capacitance is mainly in the low frequency range by using LCR meter or impedance analyzer. The measurement of the high frequency section is mainly in the research stage and the testing technology is monopolized by several countries. High frequency measurements using a vector network analyzer, specify placed the measured capacitance which constitute two port network in the appropriate location of the test system [3]. Then measure the network S parameters and then calculated other capacitance parameters from the S parameters.

Although the measurement frequency of vector network analyzer is wide, due to the effect of fixture it can often bring about measurement error. To get the parameter of the measured capacitance accurately, it is necessary to eliminate the effect of fixture [4-7]. The main techniques proposed now for eliminating harmful effects of test fixture are: modeling, embedding and direct measurement. It's especially hard to measure the parameter of the capacitance in high-frequency because of the measurement error. In addition, because the parameters of the ephemeris data are discrete, it is necessary to fit the given data to certain curve, before direct measurement is applied, to obtain the corresponding parameters of the data. Commonly used algorithms include Lagrange interpolation [13], polynomial fitting [14], cubic spline fitting [15], and cubic uniform B-spline fitting [16], etc. Many researches in this field have been

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performed in the past few years.

According to the analysis above of high frequency capacitance measurement. Based on the early research of measure method of high frequency capacitance, this paper proposed a new method which can effectively eliminate fixture effect, and obtain a accurate measured S-parameter data that combine modeling and de-embed method, the errors produced by modeling and de-embedding are corrected, and the effect of test fixture can also be accurate eliminated which produced in the testing process of the vector network analyzer. Then the performance parameters of the capacitor and other equivalent circuit parameters are calculated by using polynomial fitting method according to the S parameter which are obtained before. Experimental results show that the method we proposed can not only effectively eliminate the errors caused by the fixture effect, but also can ensure the accuracy of the measurement of the capacitance parameters in the 10GHz and even higher frequency segments.

The rest of the paper is organized as follows: The next section will discuss several curve fitting algorithms. The section III describes the proposed data fitting method. Finally, experimental results are given in section IV. Section V is the conclusion.

## 2 Data Fitting Algorithms

It is assumed the following  $m+1$  data points are given, denoted by  $(x_i, y_i)$  ( $i = 0, 1, \dots, m$ ) respectively.

### 2.1 Lagrange Interpolation

The lagrange function is defined as following:

$$P_n(x) = \sum_{k=0}^m p_k l_k(x) \quad (1)$$

where  $l_k(x)$  is the Lagrange basis function which is a polynomial of order  $m-1$ , satisfying the following condition:

$$l_k(x) = \prod_{\substack{j=0 \\ j \neq k}}^m \frac{t - t_j}{t_k - t_j} \quad (2)$$

Lagrange interpolation, a relatively simple mathematical model, is the most commonly used data fitting algorithms. However, the disadvantage of this method is that, when new points are added, all the formulas need to be rebuild, which requires a large amount of computation. Since the accuracy of Lagrange interpolation is higher in the middle part of the interval than near the ends, when Lagrange interpolation is applied, the interpolated points should be made as near as possible to the middle part of the interval.

### 2.2 Polynomials Fitting

There are three methods to think about the fitting function  $P(x)$ . Method one is the maximum absolute value  $\max_{1 \leq i \leq m} |r_i|$  of error  $r_i = p(x_i) - y_i$  ( $i = 0, 1, \dots, m$ ); Method two is to seek the sum of the absolute

error  $\sum_{1 \leq i \leq m} |r_i|$ ; Method three is to use the square root error  $\sum_{i=0}^m r_i^2$ . Among them, the first two methods are natural, simple, but not easy to perform differential operation, the method 3 is equivalent to seek the square of 2-norm, so we often use the sum of squared error  $\sum_{i=0}^m r_i^2$  to measurement error  $r_i$  ( $i = 0, 1, \dots, m$ ) in data fitting .

Specific approach of least squares fit is: seeking  $P(x) \in \Phi$ , which is in the selected class of functions  $\Phi$ , to make the sum of squared error minimum for given  $(x_i, y_i)$  ( $i = 0, 1, \dots, m$ ), that is

$$\sum_{i=0}^m r_i^2 = \sum_{i=0}^m [p(x_i) - y_i]^2 = \min \tag{3}$$

Among them, to least-squares curve fitting, there are many methods to select functions  $\Phi$ .

Assumptions to the given data points  $(x_i, y_i)$  ( $i = 0, 1, \dots, m$ ), and function  $\Phi$  for all polynomial whose orders is no more than  $n$  ( $n \leq m$ ). Now we can get  $p_n(x) = \sum_{k=0}^n a_k x^k \in \Phi$ , to make

$$I = \sum_{i=0}^m [p_n(x_i) - y_i]^2 = \sum_{i=0}^m \left( \sum_{k=0}^n a_k x_i^k - y_i \right)^2 = \min \tag{4}$$

When fitting a polynomial function it called polynomial fitting and  $P_n(x)$  satisfies the formula (4) is the least squares polynomial fitting. Particularly, when  $n = 1$ , it is called linear fitting or linear fit.

Thus, the above problem is to get the extreme value of  $I = I(a_0, a_1, \dots, a_n)$ . From the necessary conditions of the extremes, we can get

$$\frac{\partial I}{\partial a_j} = 2 \sum_{i=0}^m \left( \sum_{k=0}^n a_k x_i^k - y_i \right) x_i^j = 0, \quad j = 0, 1, \dots, n \tag{5}$$

that is

$$\sum_{k=0}^n \left( \sum_{i=0}^m x_i^{j+k} \right) a_k = \sum_{i=0}^m x_i^j y_i \quad j = 0, 1, \dots, n \tag{6}$$

Formula (6) is linear equations on  $a_0, a_1, \dots, a_n$ , with the matrix represented as

$$\begin{bmatrix} m+1 & \sum_{i=0}^m x_i & \Lambda & \sum_{i=0}^m x_i^n \\ \sum_{i=0}^m x_i & \sum_{i=0}^m x_i^2 & \Lambda & \sum_{i=0}^m x_i^{n+1} \\ M & M & \Lambda & M \\ \sum_{i=0}^m x_i^n & \sum_{i=0}^m x_i^{n+1} & \Lambda & \sum_{i=0}^m x_i^{2n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ M \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^m y_i \\ \sum_{i=0}^m x_i y_i \\ M \\ \sum_{i=0}^m x_i^n y_i \end{bmatrix} \tag{7}$$

Formula (4) or (7) is called the normal equations. The solution of formula (7) is unique. Solve equation (7) we can get  $a_k$  ( $k = 0, 1, \dots, n$ ), and thus obtain

$$p_n(x) = \sum_{k=0}^n a_k x^k \tag{8}$$

So  $P_n(x)$  is the fitting polynomial that we attempt to get.

### 2.3 Cubic Spline Fitting

For a given division  $\Delta$  of the interval  $[a, b]$ , say  $a = t_0 < t_1 < \dots < t_m = b$  and for a given set of data points  $(t_i, p_i)$  ( $i = 0, 1, \dots, n$ ), the function  $s(t)$  is said to be a cubic polynomial passing through the given  $n$  data points if the function  $s(t)$  satisfies the following three conditions:

- (1)  $s(t)$  is a polynomial of degree 3 defined in each sub-interval  $(t_i, p_i)$  ( $i = 0, 1, \dots, n$ );
- (2)  $s(t)$  is twice continuously differentiable in the whole interval  $[a, b]$ ;
- (3)  $s(t_i) = p_i$ , ( $i = 0, 1, \dots, n$ ).

### 3 Proposed Method

In this part, we proposed a new method for eliminating the error of the test fixture first; then the performance parameters of the capacitor and other equivalent circuit parameters are calculated by using polynomial fitting according to the S parameters which are obtained before.

#### 3.1 Eliminate Errors of Test Fixture

This paper proposed a new method for eliminating the error of the test fixture, which is used to measure the device under test with test fixture, and eliminate the influence of the test fixture in the measurement results, and which including the following process:

(1) Using a test fixture clamping the component to be measured; then combine the measuring pieces with the standard parts for parity that will be measured, then measure the DUT using test fixture clamped; among them, the standard is short standards devices or open standard device. And as we know, open circuit standard device is equivalent to a test fixture not used, and short circuit standard device is equivalent to the standard device that through a wire to connect all the terminals.

(2) Using automatic port extension respectively test the DUT and calibrate the DUT, then we can get test fixture parameters and measurement parameters and check parameters. Among which the automatic port extending technique is proposed by Agilent Technologies, using automatic port extension, test port from the connecting part extended to the end of the transmission line, thus eliminating the effect of fixture.

(3) Revise the DUT's parameters with the parity DUT's parameters: theory, DUT parameters is changing linearly with the parity DUT's parameters and a linear proportion which can be gotten by many experiments results, the range of the results is marked with A. But in fact, the use of automatic port extension test fixture calibration for reflect measurement method will have a lot of errors, so it is first determined whether parity DUT's parameters and DUT's parameters are in a region A, if it's in the region, the measured parameters are accurate, then end the test; otherwise, implement process4.

(4) Through measure the parameters of standard devices we can obtain S parameter matrix of the test fixture, and de-embed the results by using the matrix operation, and the combination parameters of DUT and the test fixture were obtained. Then split the combination parameters of DUT to get DUT parameters and fixture parameters; among them, de-embedding the DUT's results by using matrix operation.

(5) Through many experiments or other ways to process the measured parameters C1 from process2 and measured parameter C2 from process4 and obtain the ratio of C1 and C2 called B, the ratio of B can be value which is in a range also can be a definite value, then make ratio operation on DUT parameters and fixture parameters obtain from process4, if the ratio is in the range of B, then the parameter will not be modified, then end the test; if the ratio is not in the range of B, then modify the DUT's parameters obtained by process4.

#### 3.2 Calculate the Capacitance Parameters

By the de-embedded capacitor S parameters, we can integrate performance parameters of capacitance: capacitor value (C), the equivalent series resistance (ESR), Q value, tangent of the loss angle (TanD), the series resonant frequency, the parallel resonance frequency, equivalent circuit parameters, capacitance value (C), the equivalent series inductance (ESL), equivalent series resistance (ESR), the equivalent parallel capacitance (EPC), equivalent parallel resistance (EPR), we can get them through polynomial fitting algorithms:

According to the analysis of 2.C, the general method of polynomial fitting can be summarized as the following steps:

(1) Drawn the scatter diagram of S parameters to determine the order of the polynomial;

(2) Calculate  $\sum_{i=0}^m x_i^j (j = 0, 1, \Lambda, 2n)$  and  $\sum_{i=0}^m x_i^j y_i (j = 0, 1, \Lambda, 2n)$

(3) write out the normal equations  $a_0, a_1, \Lambda a_n$

(4) write out the fitting polynomial  $p_n(x) = \sum_{k=0}^n a_k x^k$

## 4 Experiments and Analysis

### 4.1 Experimental Procedure

Based on the method proposed above, this paper use the Labview NI development environment to build a radio frequency capacitance tester. The tester is connected by using Ni GPIB-USB-HS cable, it can automatic recognition, remote control commercial vector network analyzer, including various types of vector network analyzer of Agilent PNA Series, Agilent ENA sequences, Rohde & Schwarz Zva sequence, Rohde & Schwarz ZVB sequence. The main steps of using this tester for measuring the capacitance parameters are as follows:

- (1) Select the project path and file name.
- (2) Select the type of test capacitor, including the standard capacitor and sample capacitor. The purpose of testing the standard capacitor is to understand the testing accuracy, the working status of the calibration and calibrate the core analysis and synthesis algorithm of the calibration test instrument. Sample capacitor is a capacitor that needs to be tested for various capacitor parameters.
- (3) Select test fixture and analysis algorithm. The test fixture including coplanar waveguide test fixtures and microstrip test fixture; analysis algorithm is, described above, the least square curve fitting, polynomial series iteration and ADS equivalent circuit optimization algorithm. One of the ways in which one can accomplish the test of a capacitor.
- (4) Connect the instrument and calibration, this step includes two steps of the vector network connection and the vector network state calibration.
- (5) Test the standard capacitor, this step is mainly to test the capacitance S parameter after the de-embedding of fixture.
- (6) Test sample capacitance, we can test performance parameters of sample capacitance such as: test frequency, capacitance value, Q value, ESR value, Tan Delta, series resonant frequency and parallel resonance frequency value.

### 4.2 Results and Discuss

In this experiment, we take the ATC standard capacitor as the test sample, we test 3 different levels of standard capacitor 12pF, 82pF, 910pF at low frequency and 4 different levels of standard capacitor 1pF, 10pF, 100pF, 1000pF at high frequency. Each capacitor test 5 samples, and then remove the highest and the lowest of the 2 test values, and the remaining three test values to get the average value as an effective test data, test results are shown in table 1 and table 2.

From Table 1 we can find that all algorithms can calculate the parameters, and the algorithm we proposed the errors are smallest, and the precision are the highest. It shows that all algorithms can eliminate the effect of test fixture.

From Table 2 we can be see, not all the results are close to the standard value, but the errors of we proposed are smallest, and the precision are the highest. Since ESR is the most important parameter of the patch capacitor, and ESR is different in different frequency. Thus, this paper focuses on the measurement results of various algorithms on the 1500MHz, 5000MHz and 10000MHz frequency points (see Fig. 2 to Fig. 4). Due to the capacitance value is larger, and the smaller the ESR, 1000pf capacitors ESR measurement becomes very difficult and large capacitance value capacitor is seldom used in high frequency circuit. Therefore, this paper does not consider the ESR of the capacitor in more than 5000Hz.

From Fig. 1 to Fig. 3 we can see that in various frequency, the proposed algorithm are all better than the literature [8] and [10], the measurement values of the proposed algorithm are close to the standard capacitance value. Therefore, the accuracy of this algorithm is higher.

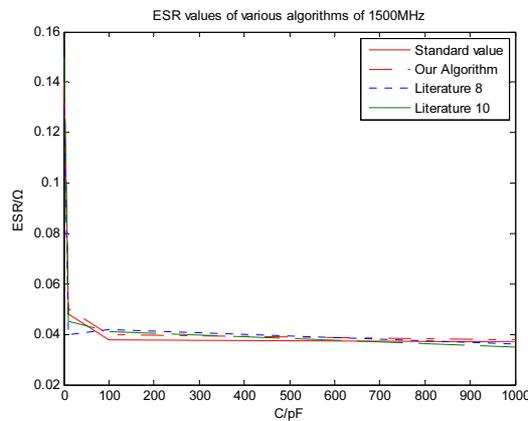
In short, compared with [8] and [10], the test data of the proposed method are compare to the standard value of ATC. Through the experimental analysis, the test data of the proposed method is most close to the standard value. It also can see that the algorithm we proposed can achieve much higher precise.

**Table 1.** Comparison of measurement data for various low-frequency capacitance parameters

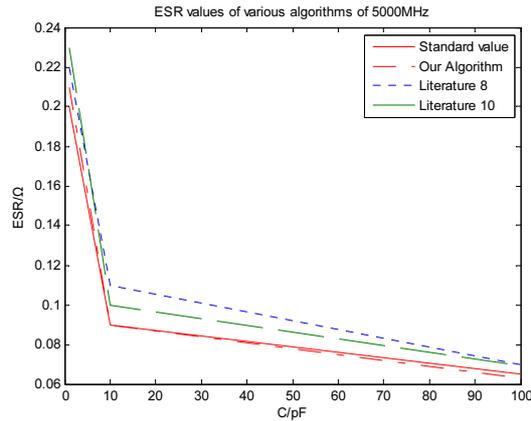
ATC	Resonant frequency	equivalent circuit parameters			ESR			
		GHz	C(pF)	L(nH)	R( $\Omega$ )	100MHz	200MHz	300MHz
12pF	standard value	1.8864	10.23	696	0.192	808	401	263
	our algorithm	1.8900	10.21	693	0.200	799	399	262
	Literature 8	1.8300	10.19	708	0.170	796	393	261
	Literature 10	1.8600	10.20	690	0.210	790	390	257
82pF	standard value	0.6619	81.9	705	0.109	174	81	47
	our algorithm	0.6713	82.0	702	0.100	175	80.5	47.5
	Literature 8	0.6814	81.20	700	0.119	176	83	48
	Literature 10	0.6900	80.30	701	0.123	172	82	48
910pF	standard value	0.1943	910	757	0.083	15.0	-	-
	our algorithm	0.1921	911	756.5	0.084	15.3	-	-
	Literature 8	0.1800	908	757.8	0.087	15.9	-	-
	Literature 10	0.1853	912	759	0.086	14.2	-	-

**Table 2.** Comparison of measurement data for various high-frequency capacitance parameters

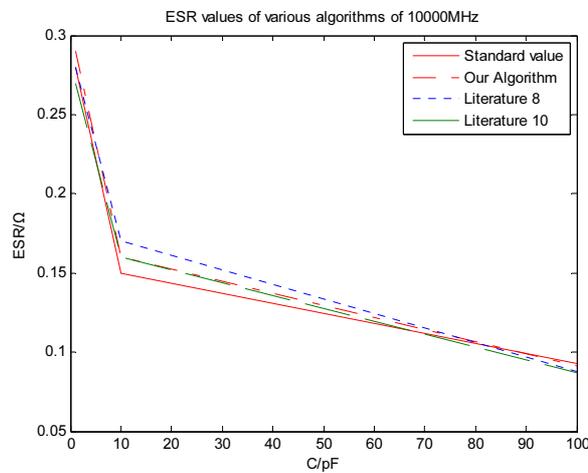
ATC	resonant frequency	equivalent circuit parameters			ESR			
		GHz	C(pF)	L(nH)	R( $\Omega$ )	1500MHz	5000MHz	10000MHz
1pF	standard value	5.413	1.12	741	0.564	0.14	0.2	0.28
	our algorithm	5.41	1.13	739	0.56	0.14	0.21	0.29
	Literature 8	5.39	1.16	748	0.57	0.13	0.22	0.28
	Literature 10	5.32	1.19	753	0.559	0.15	0.23	0.27
10pF	standard value	1.986	10.48	698	0.126	0.048	0.09	0.15
	our algorithm	1.9	10.44	700	0.13	0.05	0.09	0.16
	Literature 8	1.87	10.45	706	0.138	0.04	0.11	0.17
	Literature 10	1.855	10.44	705	0.138	0.045	0.1	0.16
100pF	standard value	642	102.54	650	0.059	0.038	0.065	0.093
	our algorithm	635	103.1	655	0.057	0.04	0.063	0.091
	Literature 8	630	104.6	680	0.065	0.042	0.07	0.988
	Literature 10	605	100.23	696	0.068	0.041	0.069	0.087
1000pF	standard value	203.56	970.9	632	0.035	0.037	-	-
	our algorithm	202.43	975.8	634	0.037	0.038	-	-
	Literature 8	200.21	978.1	639	0.039	0.036	-	-
	Literature 10	199	977.5	641	0.032	0.035	-	-



**Fig. 1.** ESR values measured by various algorithms at 1500MHz



**Fig. 2.** ESR values measured by various algorithms at 5000MHz



**Fig. 3.** ESR values measured by various algorithms at 10000MHz

## 5 Conclusions

In this paper, focusing on the problem of the measurement errors in the high frequency section are particularly prominent. We proposed a method to combine modeling and de-embedding technique to eliminate the error brought by the effect of the test fixture, and then use the polynomial fitting method to accurately integrate capacitance performance parameters and equivalent circuit parameters. Through comparing to simple use de-embedding technique for measuring capacitance parameter method, verified the method is effectiveness in 10GHz and even more high frequency. So the measurement accuracy has been effectively improved.

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