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# Angular Preference Multi-Objective Optimization Algorithms with Inverse Initialization

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**Abstract.** Since the preferred multi-objective optimization solution set is a local optimal solution with decision maker's preferences. To improve the performance of its, this paper proposes the angular preference multi-objective optimization algorithms with inverse initialization (AP-MOA). AP-MOA proposes three new strategies. The first is the target initialization strategy. There are two types of cases. For a bi-objective optimization problem, a better initial population is generated in the specified region on the preference information. For the tri-objective optimization problem, a tent mapping is used to generate uniform individuals in the specified region on the preference information. The second is two stage mutation, which is using genetic and differential mutation to produce excellent and stable offspring. The third is the angular preference guiding strategy. Two rays are drawn from the origin of the coordinates based on preference information to delineate a preferred solution region. According to experimental comparison, AP-MOA can converge quickly and obtain a satisfactory set of preference solutions.

**Keywords:** angular preference, inverse initialization, preference-based multi-objective optimization, two-stage mutation

## 1 Introduction

Preference Multi-Objective Optimization Problems (PMOP) is a prominent problem in the current field of optimization. Different from general multi-objective optimization, PMOP focuses on exploring local objective pareto regions determined based on preference information. Driven by social and technological progress, PMOP is being increasingly used in various fields. For instance, production of an assortment of commodities while optimizing profitability with restricted resources [1], calculating an appropriate range of time periods for the illumination of traffic signals [2], as well as in research [3], journey [4], generator start-up problem [5], et al..

Many algorithms of PMOP are proposed, categorized into three groups: convergence based on preference points, convergence based on preference vectors, and convergence based on individual preference points. The first category, as described in literature [6-8], redefines the optimal individual priority by using the g-dominance strategy that is based on preference points. The second category, as defined in literature [9, 10], posits that optimal individuals are preferentially selected from the preference region based on preference points and vectors, increasing individual selection pressure. The third category, as outlined in literature [11], establishes a dominance

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relation within the same pareto dominance rank by considering the proximity of individuals to the preference point.

However, current algorithms fail to consider the impact of the initial population on POMP and angular convergence optimization. This paper proposes target initialization strategy, two stage mutation and angular preference guiding strategy. It can produce better initial solutions and quickly obtain preferred solution sets.

The paper includes five sections. Section I supplies an overview of AP-MOA, PMOP, and related algorithms. Section II outlines concept of PMOP, preference region and mutation methods. Section III explains the target initialization strategy and angle preference domination strategy. Section IV conducts a comparison of the algorithms, performance analysis and summarizes the algorithm of this paper.

## 2 Related Concepts

### 2.1 PMOP

PMOP is deciding the decision maker's area of interest based on the preference information (preference point P) supplied by the decision maker, getting it to a smaller subset of the entire pareto region [12, 13]. Eventually, PMOP can be simplified to a simpler problem.

$$\begin{aligned} &\min F(x) = \min f_1(x), f_2(x), ... f_m \\ &st.\begin{cases} h_i(x) \le 0, (i = 1, \dots, p) \\ g_j(x) \le 0, (j = 1, \dots, q) \end{cases} \end{aligned} \tag{1}$$

Where $x = (x_1, x_2, \dots x_n) \in Q$ is an N-dimensional decision variable. $Q$ is the decision space. $f_i(x)$ is the $i^{th}$ objective vector in the target space $Q$, $i = 1, 2, \dots m$, and m denotes the number of objective vectors. The $q$ linear equality constraints are denoted by $g_j(x)$, and the $p$ linear inequality constraints are denoted by $h_i(x)$.

**The pareto definition of PMOP**

**Pareto dominate.** $x_y$ and $x_z$ are individual solutions. $x_y$ pareto dominates $x_z$ if the equation 2 are satisfied.

$$\begin{cases} f_i(x_y) \le f_i(x_z), \forall i = \{1, \dots, m\}. \\ f_j(x_y) < f_j(x_z), \exists j \in \{1, 2, \dots, m\}. \end{cases} \tag{2}$$

**Pareto optimal solution.** $x_z$ is a solution of PMOP. Once equation 3 has been satisfied, $x_z$ is the pareto optimal solution.

$$\neg\ \exists x \in R, x \prec x_z \tag{3}$$

**Pareto optimal solution set**. Individuals of the set do not dominate each other.

$$F = \{x \in R \mid \neg\ \exists x_z \in R, x_z \prec x\} \tag{4}$$

### 2.2 Region of Preference

**Preference vector** [14]. vector of directions from the coordinate origin to the P, displayed in Fig. 1.

**Preference area**. Euclidean distance for an individual to reach the preference vector, which does not exceed the range R (specified by the decision maker), displayed in Fig. 1. the Euclidean distance formula:

$$Dis(A|P) = |A| * \left(1 - \left(\frac{A * OP}{|A| * |OP|}\right)^2\right)^{\frac{1}{2}} \tag{5}$$

Where A is an individual and OP is a preference vector.

**Preference dominate**. Individuals of the same rank, individuals in the region of preference dominate individuals outside the region of preference. Individuals in the same region determine dominance relationships by crowding degree [10].

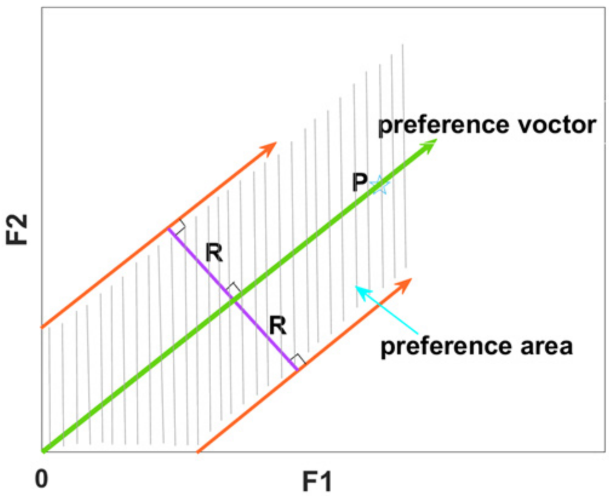



**Fig. 1.** Preference area information

## 3 AP-MOA

This section first proposes target initialization strategy, followed by angle preference dominance strategy, then two stage mutation, and finally AP-MOA structure.

### 3.1 Target Initialization Strategy

The NSGAII typically uses random initialization when initializing populations, resulting in an unstable and inherently random starting point [15, 16]. However, it is important to note that the initial population's quality significantly affects the optimization outcomes. To address it, this paper proposes implementing a targeted initialization strategy.

For problems with bi-objectives, start by initializing the population within the area that is dominated by the preference point, subsequently constraining the decision variables of each individual to a practical range. By randomly generating populations in a specific region, this paper demonstrates the ability to steer and maintain population diversity, a is rand(*) that is random function.

$$dec_1 = P_{(1)} + \left(1 - P_{(1)}\right) * a \tag{6}$$

$$b = \begin{cases} a \quad\quad\quad , \ a \le 0.5 \\ a - 0.5, \ a > 0.5 \end{cases} \tag{7}$$

$$dec_{(j)} = k_{ll_{(j)}} + \left(k_{ul_{(j)}} - k_{ll_{(j)}}\right) * b, j = 2 \ldots n \quad \textbf{(8)}$$

Where P(1) is the first value of the preference point, dec denotes individual solution vector.

As the number of objectives increases is difficult to control the scope of individual solutions. This paper proposes the inverse function construction method to solve this problem.

**Inverse function construction method**

For problems with tri-objectives, it uses a tent mapping initial strategy [17] to set the initial values of the objective function within the designated area. Subsequently, inverse function construction method is used to invert the decision variable values of the relevant individuals. This approach ensures the effective population initialization within the specified region.

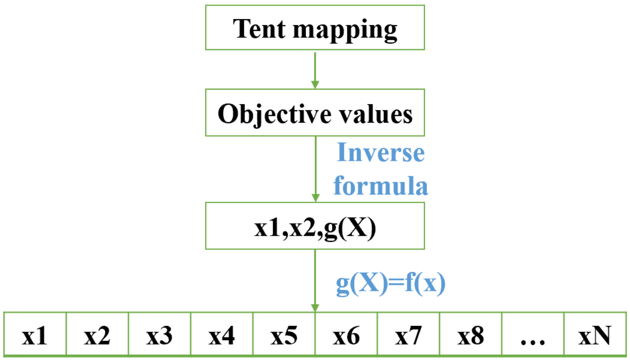



**Fig. 2.** Inverse tectonic diagram

Objective function values for individuals were produced in the spatial region. Getting inverse functions by using tri-objective inverse constructor formula, then calculating the values of $x_1$, $x_2$, and $g(X)$. The decision variables of individuals $x_i$ $(i = 1, \ldots, N)$ are calculated based on $g(X)$ and the objective function, resulting in a population for a specific region. Inverse function sets, in Table 1. The process is shown in Fig. 2. Algorithm 1 is an algorithmic implementation of target initialization strategy.

**Table 1.** Tri-objective inverse constructor formula

| Test function | Inverse formula |
|---|---|
| DTLZ2 | $\begin{cases} x_2 = \dfrac{\cot^{-1}\left(\dfrac{f_1(X)}{f_2(X)}\right)}{\pi * 0.5} \\ x_1 = \dfrac{\cot^{-1}\left(\dfrac{f_1(X)}{f_3(X)}\right)}{\cos(x_2 * \pi * 0.5)} \\ g(X) = \dfrac{f_3(X)}{\sin(x_1 * \pi * 0.5)} - 1 \end{cases}$ |

| | |
|---|---|
| DTLZ4 | $\begin{cases} x_2 = \dfrac{\cot^{-1}\left(\dfrac{f_1(X)}{f_2(X)}\right)}{(\pi*0.5)^{\frac{1}{100}}} \\ x_1 = \dfrac{\cot^{-1}\left(\dfrac{f_1(X)}{f_3(X)}\right)}{(\cos(x_2*\pi*0.5))^{\frac{1}{100}}} \\ g(X) = \dfrac{f_3(X)}{\sin(x_1^{100}*\pi*0.5)} - 1 \end{cases}$ |

**Algorithm 1.** Target initialization strategy

**Input:**
Preference point P
Population size pop
Decision variables x_num
Upper and lower limits of decision variables x_max, x_min
**Output:**
Chromo

**Procedures:**
// tri-objectives
**Step 1:** tent mapping initialize objective values
**Step 2:** for i from 1 to pop
**Step 3:** using the formulae from Table 1, compute the decision variables $x_1$, $x_2$, and g(X), using the g(X) to compute the decision variables
**Step 4:** end for
// bi-objectives
**Step 5:** for i from 1 to pop
**Step 6:** calculate dec, according to equation (6)(7)(8), using the dec to get individual
**Step 7:** chromo U individual //merge
**Step 8:** end for

### 3.2 Angle Preference Dominance Strategy

Definition 1 (Individual preference angle). Hypothetically, preference vector L, a vector M from the coordinate origin O to an individual $x_z$, the angle between the vector L and the vector M is defined as the preference angle of the individual $x_z$, denoted as angle_$x_z$|p.

$$angle\_x_z \Big| p = a\cos\left\{\frac{dot(Ox_z, OP)}{norm(Ox_z)*norm(OP)}\right\} \tag{9}$$

Where acos(*) is the cos(*) inverse function, dot(*) function is used to calculate the dot product of two vectors. norm(*) is Calculate the distance between two points as a function of the difference between the elements of the vector.

Definition 2 (Angle preference area). The vector OP intersects at preference point P and is perpendicular to the line segment, which is R long. The shaded area is the angle preference area, shown in Fig. 3.

Definition 3 (Preference angle). In Fig. 3, the angle of the gray section corresponding to the origin is the angle preference region.

$$pre_{angle} = a\sin\left\{\frac{R}{sqrt\left(P(1)^2 + P(2)^2\right)}\right\} \quad \textbf{(10)}$$

Where asin(*) is the sin(*) inverse function. In two-dimensional coordinates, for example, P is denoted as P(1, 2), P(1) is the first value of P and P(2) is the second value of P.

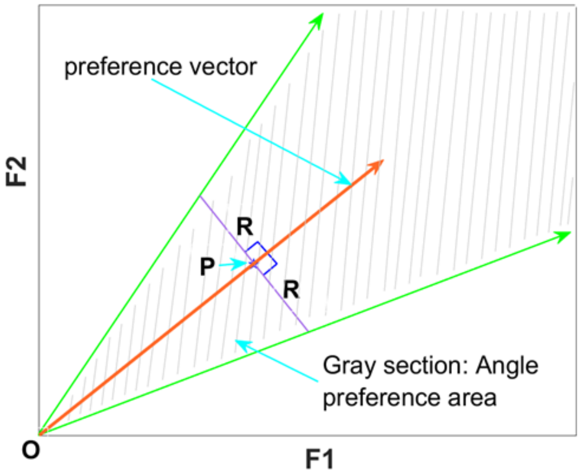



**Fig. 3.** Angle preference information

As PMOP is not an entire region for optimization. Consequently, this paper suggests using the concepts of priority search. Individuals within the region of angular preference are selected with preference as superior offspring. The specific principles are as follows: individuals within the angular preference region dominant over external individuals, individuals within the same angular region are ordered using pareto non-dominance, individuals within the same level of preference region dominate individuals outside the preference region, and individuals within the same region are prioritized based on the degree of crowding. Defining it as angle preference dominance strategy (Ap-dominate). Algorithm 2 is division of angle preference area. Algorithm 3 is an algorithmic implementation of Ap-dominate.

**Algorithm 2.** Division of angle preference area

**Input:**
mixed population input_chromo
**Output:**
chromo1
chromo2

**Procedures:**
**Step 1:** Calculate pre_angle
**Step 2:** for each $x_j \in$ input_chromo, do
**Step 3:** calculate the angle_$x_j$|p.
**Step 4:** if angle_$x_j$|p <= pre_angle
**Step 5:** chromo1 ∪ { chromo($x_j$)} // add individual
**Step 6:** else
**Step 7:** chromo2 ∪ { chromo($x_j$)}
**Step 8:** end if
**Step 9:** end for

**Algorithm 3.** Ap-dominate

**Input:**
Population size pop
Population in_chromo
**Output:**
Chromo

**Procedures:**
**Step 1:** Algorithm2 to in_chromo, get chromo1, chromo2
**Step 2:** non domination sort
**Step 3:** if num(chromo1)<=pop then
**Step 4:** num1=pop-num(chromo1)
**Step 5:** algorithm1 sort chromo2, the first num1 from chromo2 saves in temp1
**Step 6:** chromo = temp1 U chromo1 //merge
Step 7: else
**Step 8:** use Algorithm1 sort chromo1, then the first N from chromo1 saves in chromo
**Step 9:** end if

### 3.3 Two-stage Mutation

This paper proposes a two-stage mutation strategy to maintain algorithm optimality and diversity. Due to the high prominence of binary crossover and mutation in generating diverse solutions and the effectiveness of differential mutation in achieving optimal solutions, a creative integration of both methods can significantly enhance the performance of algorithmic optimization searches.

**Difference mutation**

Difference mutation operators are able to maintain the good genes of their parents during algorithmic iterations and have good algorithmic stability [18]. The formula is as follows:

$$V(g+1) = X_{r_1}(g) + F * \left(X_{r_2}(g) - X_{r_3}(g)\right) \tag{11}$$

$$F = F_0 * 2^{e^{\left(1-\frac{G_{\max}}{G_{\max}+1-G_{now}}\right)}} \tag{12}$$

Where $F_0$ is the original mutation factor, F represents the mutation factor, $x(g)$ is parent population. The individual $V(g+1)$ is variant.

Crossover of the $g^{th}$ generation population $x(g)$ and its mutant individuals $V(g+1)$:

$$V_{i,j}(g+1) = \begin{cases} V_{i,j}(g+1), & if\ a \le cr\ or\ j = j_r \\ x_{i,j}(g), & otherwise \end{cases} \tag{13}$$

where the probability of crossover is represented as cr and $j_r$ is a random integer, a is rand(*).

**Binary crossings and mutation**

Binary crossover mutations use random numbers to generate genetic individuals that can maintain population diversity [19]. The formula is as follows:

$$\begin{cases} C_1 = \frac{1}{2} * \left(1+\beta\right) * P1 + \frac{1}{2} * \left(1-\beta\right) * P2 \\ C_2 = \frac{1}{2} * \left(1-\beta\right) * P1 + \frac{1}{2} * \left(1+\beta\right) * P2 \end{cases} \tag{14}$$

$$\beta = \begin{cases} (2 * a^{\frac{1}{\mu_1+1}}, & a < 0.5 \\ \left(\frac{1}{2*\left(1-a\right)}\right)^{\frac{1}{\mu_1+1}}, & a \geq 0.5 \end{cases} \tag{15}$$

$$\delta_j = \begin{cases} (2 * a)^{\frac{1}{\mu_2+1}-1}, & a < 0.5 \\ 1-\left(2*\left(1-a\right)\right)^{\frac{1}{\mu_2+1}}, & a \geq 0.5 \end{cases} \tag{16}$$

$$\begin{cases} C_{1(j)} = C_{1(j)} + \delta_j \\ C_{2(j)} = C_{2(j)} + \delta_j \end{cases} \tag{17}$$

where C1, C2 are the offspring, while P1 and P2 represent the parents, yita1 is a parameter. yita2 is a parameter, a is rand(*). Algorithm 4 is two-stage mutation.

**Algorithm 4.** Two-stage mutation

**Input:**
Population chromo3
$F_0$=0.5, pop
**Output:**
off_spring

**Procedures:**
**Step 1:** algorithm 3 to chromo3, chromo getting it a half
**Step 2:** binary crossings and mutation gets offspring
**Step 3:** chromo4 = chromo U offspring //merge
**Step 4:** using chromo4 to difference mutation get off_spring

### 3.4 AP-MOA Algorithm

AP-MOA utilizes a novel initialization approach, incorporating two stage mutation to enhance the variety of desired individuals. This is followed by preference convergence through population employing an angular preference strategy. Algorithm 5 is AP-MOA.

**Algorithm 5.** AP-MOA

**Input:**
gen, P, R
Populations pop
**Output:**
Chromo

**Procedures:**
**Step 1:** initialize with algorithm 1 to get chromo
**Step 2:** for i from 1 to gen //start iteration
**Step 3:** preference dominate to chromo
**Step 4:** chromo = first pop/2 of chromo
**Step 5:** Algorithm 4 to get off_spring
**Step 6:** combine_chromo = off_spring U chromo //merge
**Step 7:** end for

## 4 Experiments and Analysis

This section first describes the comparison algorithms, parameter settings and performance evaluation metrics. Then compare the GD, IGD and SP performance of the algorithms. Lastly analyzes target initialization strategies and parameter sensitivity.

### 4.1 Preparations

The paper focuses on preference multi-objective optimization algorithms. it chooses g-NASGAII guided by preference points, r-NSGAII established a dominance relation and AP-MOA to participate in the comparison. The parameters are set with reference to the algorithmic optimal parameters of most papers. r-NSGAII have been assigned average weights and a non-domination threshold $\delta$ of 0.1 [20]. The AP-MOA algorithm sets the preference radius R to 0.1, $F_0$ is 0.5, cr is 0.3. The binary crossover parameter yita1 is 2 and the polynomial variation parameter yita2 is 5. The crossover and mutation probabilities are set uniformly at 0.99 and 0.1, respectively. The experimental environment is really matlab environment carried out.

We have set three types of preference points for each test function. These preference points, selected respectively from the feasible domain, real pareto frontier and infeasible domain, comprehensively evaluate the performance of the algorithms. With reference to the preference point setting of most papers, the preference points for test functions in Table 2.

**Table 2.** Function preference points and experimental parameters

| Test function | Infeasible domains | Pareto surface | Feasible domains | Pop | Iterations | Decision dimension |
|---|---|---|---|---|---|---|
| ZDT1 | (0.1, 0.2) | (0.50, 0.30) | (0.8, 0.8) | 100 | 200 | 30 |
| ZDT2 | (0.2, 0.4) | (0.60, 0.64) | (0.9, 0.9) | 100 | 200 | 30 |
| ZDT3 | (0.2, 0.2) | (0.24, 0.28) | (0.5, 0.6) | 100 | 200 | 30 |
| ZDT6 | (0.3, 0.2) | (0.60, 0.64) | (0.7, 0.8) | 100 | 200 | 10 |
| DTLZ2 | (0.2, 0.3, 0.4) | (0.50, 0.70, 0.50) | (0.7, 0.8, 0.8) | 200 | 300 | 10 |
| DTLZ4 | (0.3, 0.4, 0.5) | (0.50, 0.50, 0.70) | (0.6, 0.6, 0.8) | 200 | 300 | 10 |

This paper next describes the performance evaluation metrics GD, IGD and SP, respectively.

(1) GD: It measures how close the resulting pareto frontier P is to the optimal pareto frontier P*, for evaluating the convergence of the solution set [21]. The smaller its value, the better the algorithm performs

$$GD(P,P^*)=\frac{\sqrt{\sum_{y\in P}\min_{x\in P^*} d(x,y)^2}}{|P|} \tag{18}$$

Where P represents the optimal pareto solution set obtained via the algorithm, P* refers to the true pareto set of solutions, d(x,y) indicates the Euclidean distance between individuals x and y, and |P| denotes the number of individuals present in P.

(2) IGD: In contrast to GD, IGD can evaluate the convergence and distribution of solution sets. The smaller the IGD, the better the convergence and distribution of the solution set.

$$IGD(P^*,P)=\frac{\sum_{x\in P^*}\min_{y\in P}\left(d(x,y)\right)}{|P^*|} \tag{19}$$

where P* is the reference set and |P*| is the number of individuals in P*. min(d(x, y)) is the minimum Euclidean distance from the individual that connects P* to P.

(3) SP: SP measures the distributability of the solution set. It is the standard deviation of the minimum distance from each solution to the other solutions and is used as an important measure of the difference between neighboring solutions in each range. The smaller the value, the more uniform the solution set.

$$SP = \frac{1}{m-1}\sum_{i=1}^{m}\left(d_i - \overline{d}\right)^2 \quad \textbf{(20)}$$

where m represents the number of individuals in the solution set, $d_i$ represents the minimum Euclidean distance from the $i^{th}$ solution to the other solutions, and $\overline{d}$ represents the average of all $d_i$.

### 4.2 Performance Comparison

Taking the mean and standard deviation of 30 evaluations of the indicator as a measure. And this paper will be divided into three categories: feasible, fronts, and infeasible area, to conduct experiments. Optimal data is highlighted through bolding.

**Table 3.** Results (mean and standard deviation) of the GD in the feasible domain for the preference point

| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 1.936e-4 | 3.301e-5 | 1.262e-4 | 1.200e-4 | **2.486e-5** | **1.526e-6** |
| ZDT2 | 2.097e-4 | 5.103e-5 | 1.813e-4 | 2.280e-4 | **2.227e-5** | **2.493e-6** |
| ZDT3 | **5.814e-5** | 1.180e-5 | 1.675e-3 | 3.101e-3 | 2.704e-4 | **8.702e-6** |
| ZDT6 | 2.743e-4 | 3.201e-4 | 1.624e-4 | 9.802e-5 | **1.617e-4** | **5.560e-6** |
| DTLZ2 | 5.191e-4 | 2.800e-5 | 5.767e-4 | 6.409e-5 | **2.236e-4** | **7.378e-6** |
| DTLZ4 | 4.515e-4 | **9.502e-6** | 5.379e-4 | 1.490e-5 | **3.696e-4** | 2.315e-5 |

**Table 4.** Results (mean and standard deviation) of the GD in the fronts for the preference point

| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 8.378e-5 | 1.945e-4 | 2.118e-5 | 2.223e-5 | **1.038e-5** | **3.237e-6** |
| ZDT2 | 1.206e-3 | 2.845e-3 | 1.296e-5 | 7.158e-6 | **1.022e-5** | **3.139e-6** |
| ZDT3 | 3.094e-4 | 1.200e-3 | **1.840e-5** | 9.645e-6 | 2.115e-4 | **8.602e-6** |
| ZDT6 | 3.528e-2 | 6.667e-2 | 1.519e-4 | 8.501e-5 | **1.448e-4** | **6.065e-6** |
| DTLZ2 | 6.461e-4 | 7.766e-4 | 4.630e-4 | 1.467e-5 | **2.398e-4** | **1.044e-5** |
| DTLZ4 | 4.810e-4 | 1.201e-4 | 4.862e-4 | **1.401e-5** | **3.601e-4** | 2.382e-5 |

**Table 5.** Results (mean and standard deviation) of the GD in the infeasible domain for the preference point

| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 2.837e-4 | 2.867e-4 | 6.189e-5 | 1.123e-4 | **2.183e-5** | **3.190e-6** |
| ZDT2 | 4.240e-4 | 4.689e-4 | 4.988e-5 | 4.978e-5 | **3.585e-5** | **3.290e-6** |
| ZDT3 | 2.768e-4 | 1.000e-3 | **5.784e-5** | 7.545e-5 | 2.820e-4 | **7.821e-6** |
| ZDT6 | 4.067e-4 | 2.334e-4 | 1.400e-4 | 7.867e-5 | **1.040e-4** | **6.169e-6** |
| DTLZ2 | 3.035e-3 | 1.956e-3 | 4.858e-4 | 3.523e-5 | **2.535e-4** | **1.658e-5** |
| DTLZ4 | 1.268e-3 | 7.434e-4 | 5.001e-4 | **1.112e-5** | **4.089e-4** | 2.314e-5 |

**Table 6.** Results (mean and standard deviation) of the IGD in the feasible domain for the preference point

| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 2.876e-2 | 2.088e-4 | 2.759e-1 | 2.506e-2 | **6.541e-4** | **1.687e-5** |
| ZDT2 | 6.876e-2 | 5.434e-4 | 3.627e-1 | 3.876e-2 | **8.022e-4** | **3.000e-5** |
| ZDT3 | 1.854e-1 | 3.378e-3 | 4.338e-1 | 4.067e-2 | **2.200e-3** | **8.150e-5** |
| ZDT6 | 5.392e-1 | 2.429e-2 | 1.915e-1 | 1.378e-2 | **1.400e-3** | **5.880e-5** |
| DTLZ2 | 2.513e-1 | 2.329e-3 | 4.930e-1 | 1.256e-2 | **3.000e-3** | **1.200e-4** |
| DTLZ4 | 3.841e-1 | 5.249e-4 | 4.585e-1 | 9.289e-2 | **1.020e-3** | **2.490e-4** |

**Table 7.** Results (mean and standard deviation) of the IGD in the fronts for the preference point

| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 3.146e-1 | 1.172e-3 | 3.107e-1 | 3.356e-3 | **6.175e-4** | **3.945e-5** |
| ZDT2 | 3.698e-1 | 4.567e-3 | 3.672e-1 | 3.006e-3 | **7.952e-4** | **3.379e-5** |
| ZDT3 | 4.772e-1 | 9.689e-3 | 4.690e-1 | 6.678e-3 | **2.500e-3** | **9.558e-5** |
| ZDT6 | 2.840e-1 | 1.101e-2 | 2.167e-1 | 6.612e-3 | **1.200e-3** | **8.701e-5** |
| DTLZ2 | 5.363e-1 | 1.810e-2 | 5.417e-1 | 2.800e-3 | **3.200e-3** | **1.448e-4** |
| DTLZ4 | 5.413e-1 | 1.902e-2 | 4.925e-1 | 6.900e-3 | **1.110e-3** | **1.101e-4** |

**Table 8.** Results (mean and standard deviation) of the IGD in the infeasible domain for the preference point

| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 7.597e-2 | 1.145e-2 | 3.467e-1 | 2.503e-2 | **7.209e-4** | **2.700e-5** |
| ZDT2 | 7.118e-2 | 1.400e-3 | 3.317e-1 | 2.579e-2 | **6.564e-4** | **3.706e-5** |
| ZDT3 | 2.994e-1 | 3.304e-3 | 4.428e-1 | 1.667e-2 | **2.300e-3** | **1.201e-4** |
| ZDT6 | 1.913e-2 | 1.000e-3 | 3.448e-1 | 1.001e-2 | **8.200e-4** | **3.700e-5** |
| DTLZ2 | 1.970e-1 | 4.107e-3 | 5.184e-1 | 2.104e-2 | **3.500e-3** | **2.003e-4** |
| DTLZ4 | 2.887e-1 | 2.801e-3 | 4.435e-1 | 1.106e-2 | **1.700e-3** | **3.105e-4** |

**Table 9.** Results (mean and standard deviation) of the SP in the feasible domain for the preference point

| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 6.092e-1 | 3.718e-2 | 1.030e+0 | 2.303e-2 | **3.396e-4** | **3.302e-5** |
| ZDT2 | 6.717e-1 | 3.201e-2 | 1.034e+0 | 3.106e-2 | **4.935e-4** | **6.301e-5** |
| ZDT3 | 8.214e-1 | 1.815e-2 | 1.053e+0 | 5.605e-2 | **4.828e-4** | **4.809e-5** |
| ZDT6 | 8.518e-1 | 6.223e-2 | 9.067e-1 | 2.109e-2 | **8.364e-4** | **6.505e-5** |
| DTLZ2 | 6.763e-1 | 3.100e-2 | 9.767e-1 | 1.902e-2 | **2.600e-3** | **1.505e-4** |
| DTLZ4 | 8.241e-1 | 2.101e-2 | 8.725e-1 | 4.200e-2 | **3.000e-3** | **1.504e-4** |

**Table 10.** Results (mean and standard deviation) of the SP in the fronts for the preference point

| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 9.980e-1 | 1.521e-3 | 9.907e-1 | 4.101e-3 | **7.222e-4** | **8.510e-5** |
| ZDT2 | 1.004e+0 | 6.690e-3 | 9.988e-1 | 3.204e-3 | **7.064e-4** | **7.240e-5** |
| ZDT3 | 1.004e+0 | 1.660e-2 | 9.992e-1 | 4.902e-3 | **6.867e-4** | **7.080e-5** |
| ZDT6 | 1.158e+0 | 2.770e-1 | 9.332e-1 | 1.501e-2 | **1.000e-3** | **7.420e-5** |
| DTLZ2 | 9.925e-1 | 2.720e-2 | 9.756e-1 | 6.301e-3 | **3.400e-3** | **1.660e-4** |
| DTLZ4 | 9.968e-1 | 1.160e-2 | 9.162e-1 | 1.507e-2 | **3.500e-3** | **1.580e-4** |

**Table 11.** Results (mean and standard deviation) of the SP in the infeasible domain for the preference point

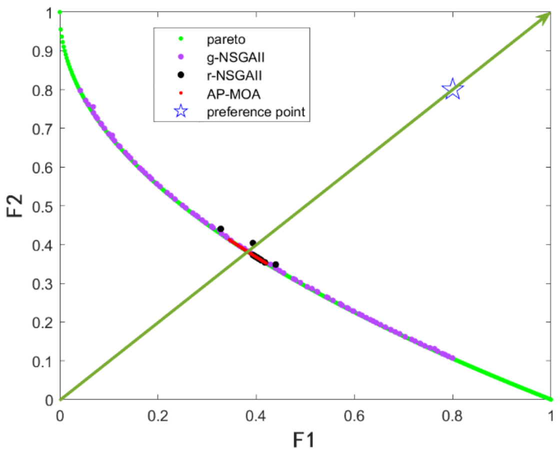
| Test functions | g-NSGAII | | r-NSGAII | | AP-MOA | |
|---|---|---|---|---|---|---|
| | Mean | Std | Mean | Std | Mean | Std |
| ZDT1 | 7.436e-1 | 3.414e-2 | 1.014e+0 | 1.526e-2 | **7.539e-4** | **1.314e-4** |
| ZDT2 | 7.134e-1 | 2.227e-2 | 1.020e+0 | 2.317e-2 | **7.480e-4** | **9.114e-5** |
| ZDT3 | 9.203e-1 | 1.526e-2 | 1.010e+0 | 1.711e-2 | **5.158e-4** | **5.413e-5** |
| ZDT6 | 5.418e-1 | 4.413e-2 | 9.143e-1 | 1.014e-2 | **9.784e-4** | **9.226e-5** |
| DTLZ2 | 6.744e-1 | 4.311e-2 | 1.001e+0 | 2.119e-2 | **3.300e-3** | **2.318e-4** |
| DTLZ4 | 7.371e-1 | 2.619e-2 | 8.453e-1 | 2.524e-2 | **3.400e-3** | **1.824e-4** |

For the convergence performance of the algorithm. From Table 3, on the feasible domain, AP-MOA exhibits superior convergence on ZDT1 and ZDT2 and marginally better performance than the other two algorithms on ZDT6, DTLZ2, and DTLZ4. However, it proves a weaker convergence effect than g-NSGAII on ZDT3. From Table 4, on the frontier, AP-MOA exhibits exceptional convergence performance on ZDT1 and ZDT2, and convergence effect outperforms the other two algorithms on ZDT6, DTLZ2, and DTLZ4. However, on ZDT3, its convergence is inferior to that of r-NSGAII. From Table 5, in infeasible domain. AP-MOA exhibits significantly better convergence performance on both ZDT1 and ZDT2, and better on ZDT6, DTLZ2, and DTLZ4, but its convergence results are inferior to r-NSGAII's on ZDT3.

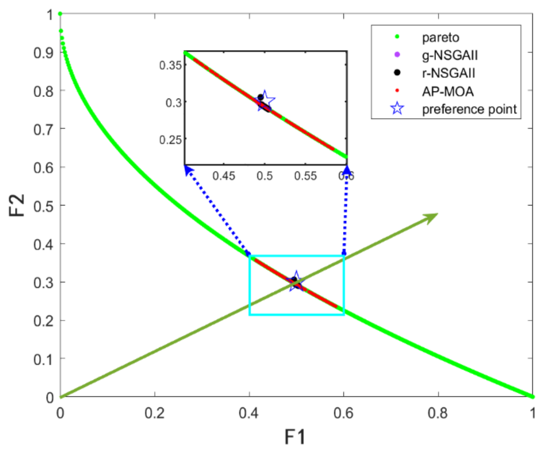
Table 6 to Table 8 show the distributability and convergence performance of algorithms. The superior performance of AP-MOA over other algorithms proves its exceptional distribution and convergence.

Table 9 to Table 11 display the distributability of the AP-MOA algorithm. Based on the table data, AP-MOA displays superior distributivity compared to the other algorithms, showing a high quality of the solution set.

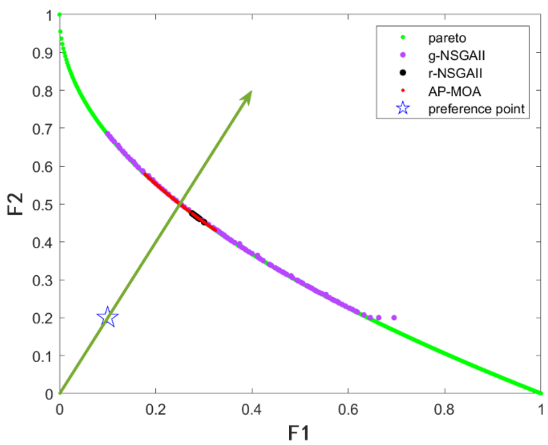
By comparing the three indicators mentioned above, the robustness of AP-MOA is good. Based on the analysis and comparison of three indicators, AP-MOA demonstrates high distributability and convergence, yielding signal outcomes. The positioning of various preference points minimally affects AP-MOA, displaying its impressive stability.



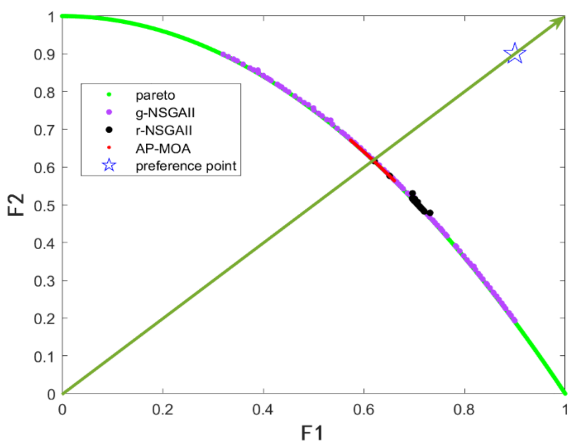

ZDT1 (0.8, 0.8)



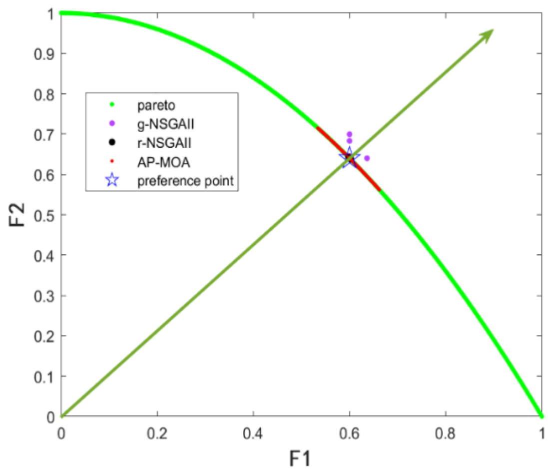

ZDT1 (0.5, 0.3)



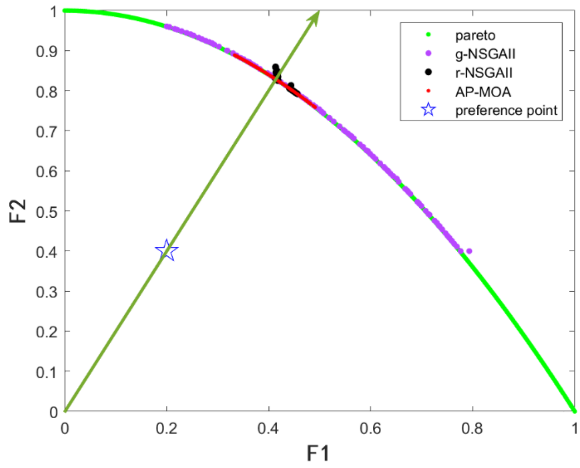

ZDT1 (0.1, 0.2)



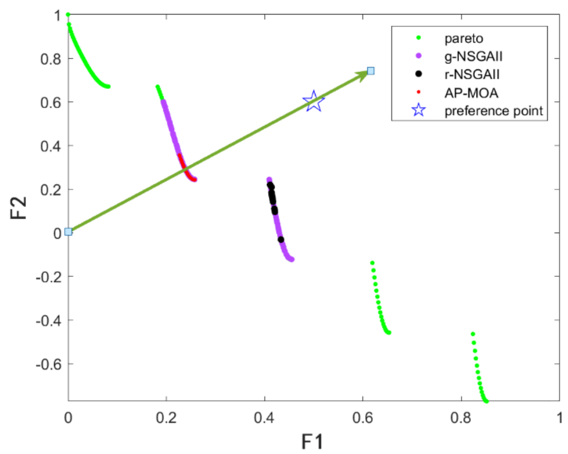

ZDT2 (0.9, 0.9)



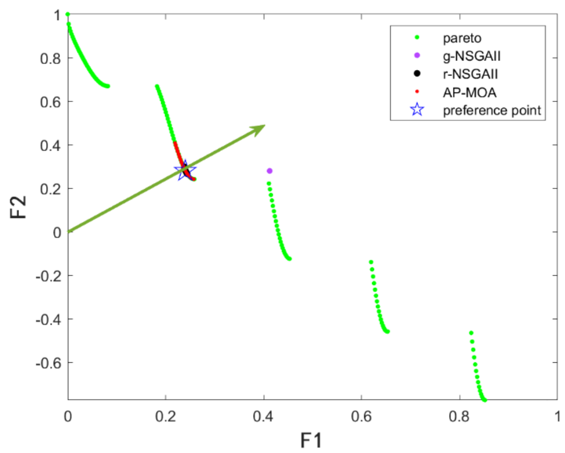

ZDT2 (0.6, 0.64)



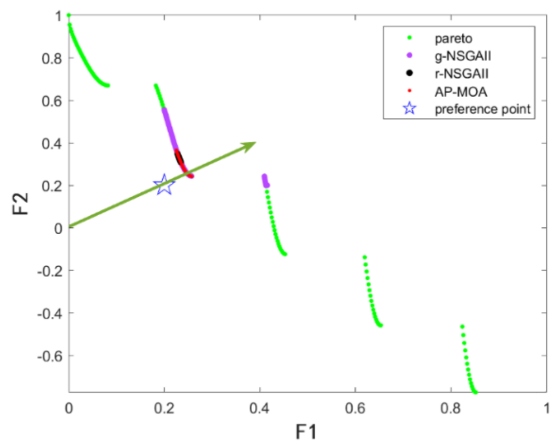

ZDT2 (0.2, 0.4)



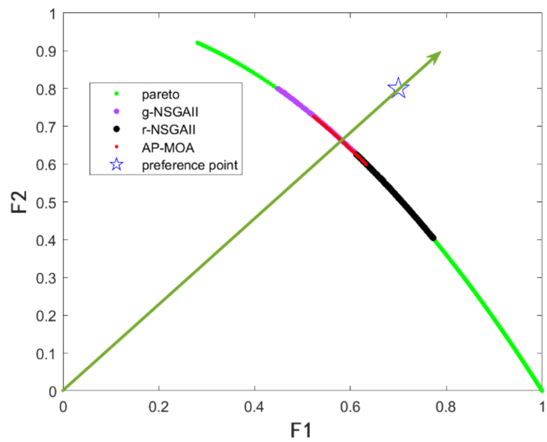

ZDT3 (0.5, 0.6)



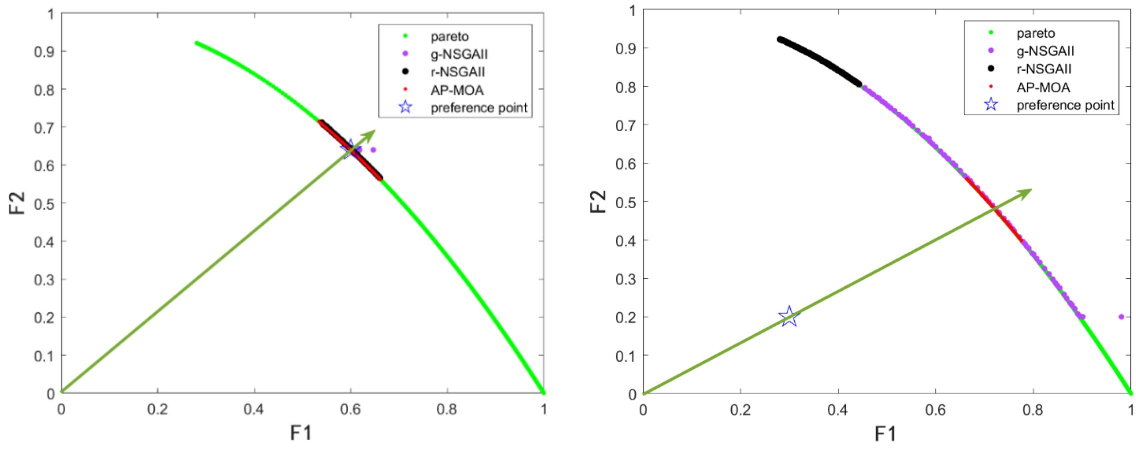

ZDT3 (0.24, 0.28)




ZDT3 (0.2, 0.2)



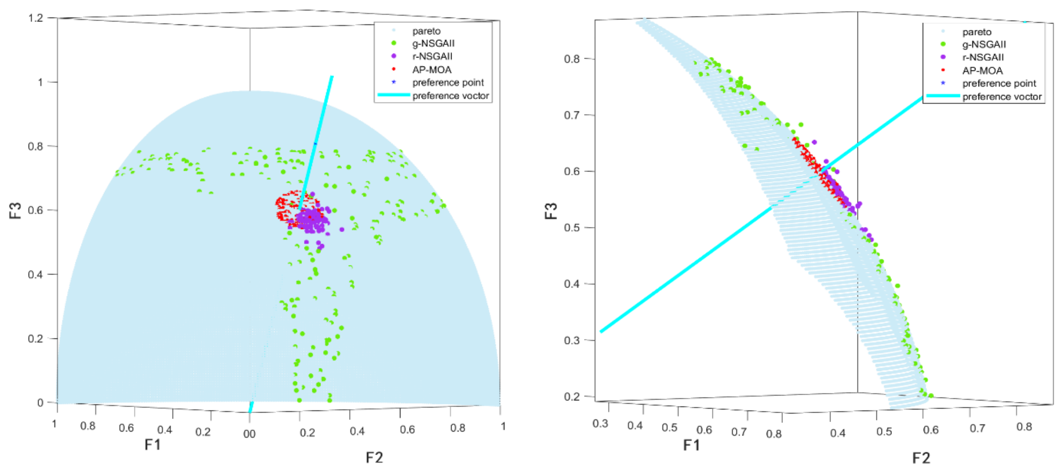

ZDT6 (0.7, 0.8)



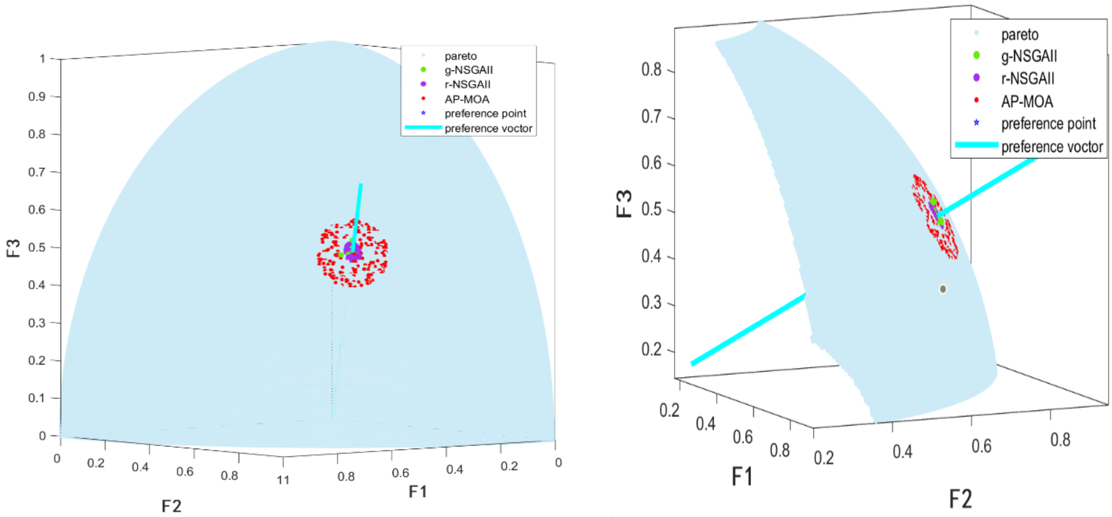

ZDT6 (0.6, 0.64) ZDT6 (0.3, 0.2)

**Fig. 4.** The effect of the three algorithms on the ZDT function at each preference point

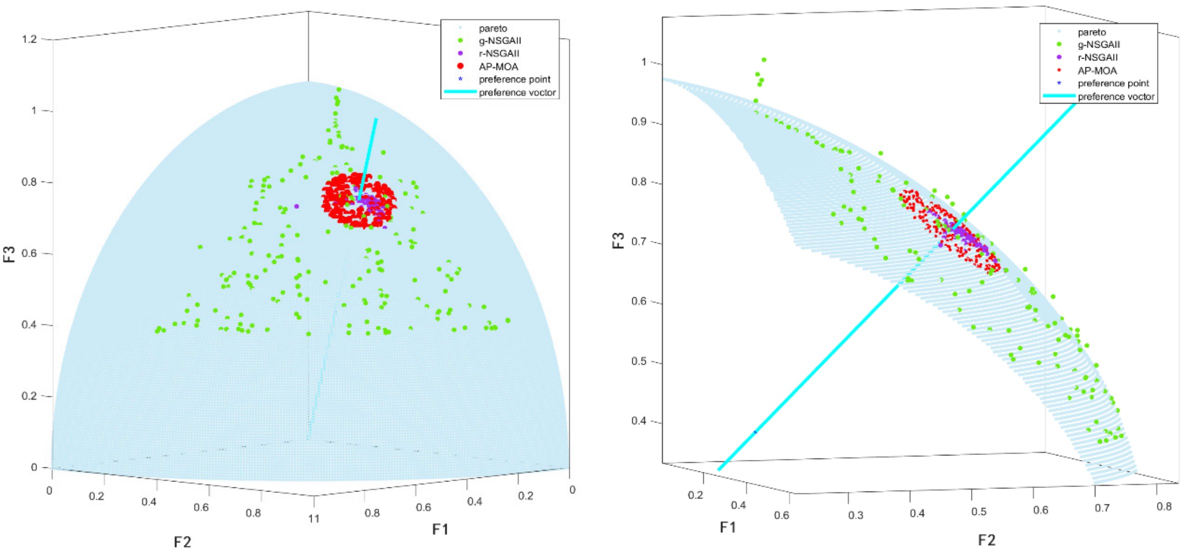
From Fig. 4, the AP-MOA algorithm can get a solution set that is of the right magnitude and coincides with a the pareto, thereby facilitating a prompt selection of the optimal solution by the decision-maker.




DTLZ2 (0.7, 0.8, 0.8) Side view




DTLZ2 (0.5, 0.7, 0.5) Side view

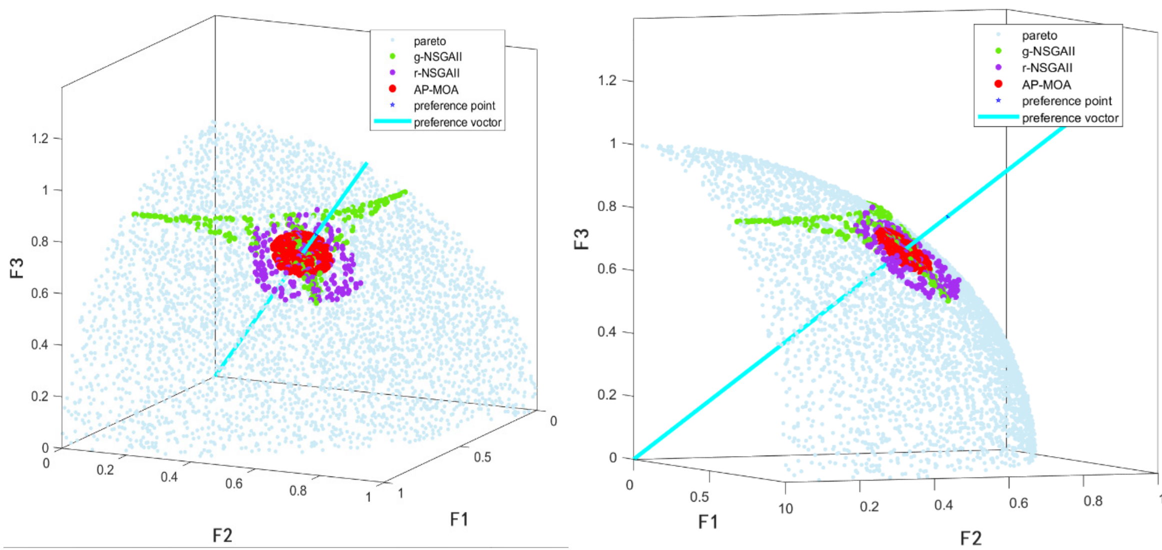



DTLZ2 (0.2, 0.3, 0.4) Side view

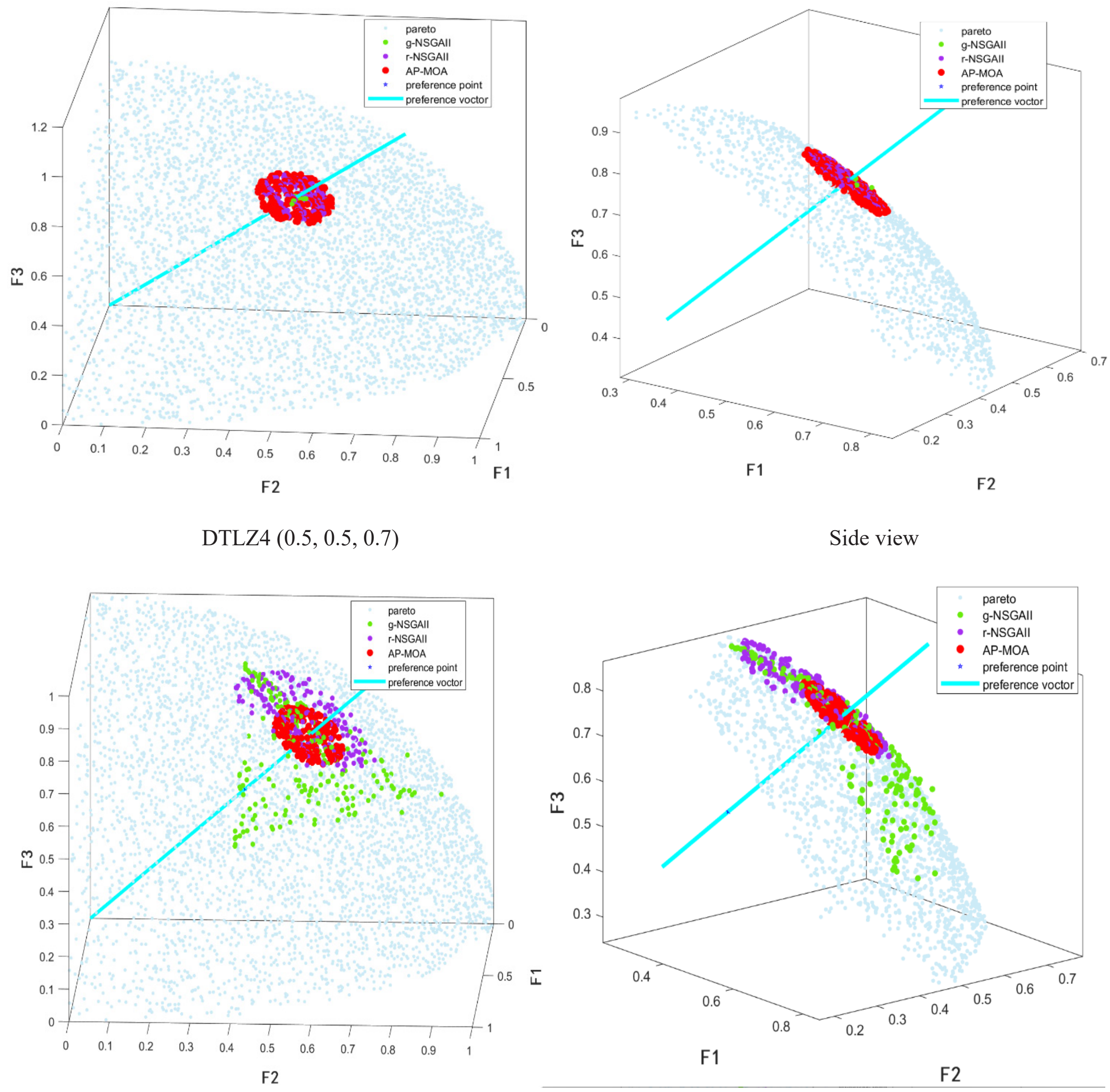
**Fig. 5.** Plot of the effect of the three algorithms on the DTLZ2 function for each preference point

From the diagram depicted in Fig. 5, the AP-MOA algorithm can procure a solution set that is uniformly distributed and coincides with pareto in DTLZ2, with well-represented preferences.

From Fig. 6, the AP-MOA algorithm can achieve a uniformly distributed set of solutions coinciding with a pareto in DTLZ4, effectively representing the preference.




DTLZ4 (0.6, 0.6, 0.8) Side view




DTLZ4 (0.5, 0.5, 0.7)

Side view

DTLZ4 (0.3, 0.4, 0.5)

Side view

**Fig. 6.** Plot of the effect of the three algorithms on the DTLZ4 function for each preference point

## 4.3 Target Initialization Strategy Analysis

In the initialization of the classical NSGAII algorithm, the rand function is used to supply random initialization [22]. This initialization type is characterized by random uncertainty, instability, and a lack of goal-directedness.

The target initialization strategy and the angle preference dominance strategy are analyzed in this subsection. By control variable method, only the location (classification of dominant and non-dominant area by preference points) where the initial individuals are generated being altered, calculating the mean of 30 GDs. Using ZDT1 as an experimental function, and the reference point is (0.5, 0.3).

**Table 12.** GD for initial strategy comparison (The top three figures are showed in the top right-hand corner)

| Area where the initial individual is located | Range of decision variables | Mean | Std |
|---|---|---|---|
| Dominated area | (0,1) | **2.2760e-05**[II] | **2.9779e-06**[II] |
| Dominated area | (0,0.5) | **2.0397e-05**[I] | **1.1137e-06**[I] |
| Randomly area | (0,1) | 2.5083e-05 | 6.6657e-06 |
| Randomly area | (0,0.5) | **2.3014e-05**[III] | 4.4730e-06 |
| Non-dominated area | (0,1) | 2.7023e-05 | 6.3740e-06 |
| Non-dominated area | (0,0.5) | 2.4008e-05 | **3.5665e-06**[III] |

The initialization strategy presented in Table 12 can enhance performance, Fig. 7 shows a comparison of the effect. The analysis shows that initial targets clustered in the dominant region yield outstanding offspring. Additionally, restricting the range of decision variables can reduce the time required for offspring to reach the frontier. In conclusion, the target initialization strategy improves algorithm performance.

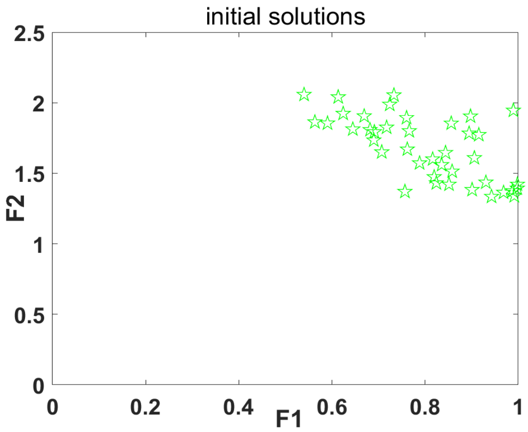



**Fig. 7.** Initial effect comparison chart

### 4.4 Parameter Sensitivity Analysis

The setting of R affects the size of the range of preference solution sets and the quality of optimization. This subsection analyses the effect of the range R of the preference region on the performance of the algorithm. The algorithm's performance is influenced by the R, as the angular preference region is dependent on this value. Assess the performance of the algorithm with R increased in increments of 0.05 over a range of 0.05 to 0.35. The average of 30 GDs was used as the measurement parameter.

**Table 13.** Mean values of 30 GDs at ZDT1 (0.5, 0.3)

| R | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 |
|---|---|---|---|---|---|---|---|
| GD | 2.0482e-5 | **2.0015e-5** | 2.1104e-5 | 2.0456e-5 | **2.0013e-5** | 2.2240e-5 | 2.1969e-5 |

From Table 13, the performance is better at $R = 0.25$ and 0.1, but $R = 0.25$ will make the solution distribution range too large, not conducive to decision-making, the comparison is shown in Fig. 8. So, $R = 0.1$ is more suitable.

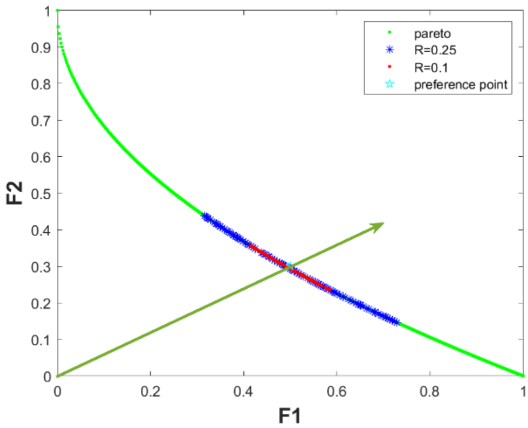



**Fig. 8.** Effect graphs for R=0.25 and 0.1

### 4.5 Summary

In this paper, AP-MOA has been proposed. Firstly, the target initialization strategy can produce a better initial population, then the two stage mutation can generate better offspring, lastly, the angular preference guiding strategy can improve optimization and convergence. When facing a preference-based multi-objective optimization problem, AP-MOA is capable of accurately reflecting the decision maker's preferences and obtaining the optimal solution set within the specified preference region. This significantly reduces the onerous selection pressure that decision makers typically face.

Regarding the application of AP-MOA for resource scheduling of mobile intelligence and mobile terminals at the edge, it can prioritize the latency objective and reduce the waiting time for user services, thus improving the user's sense of experience. Consequently, PMOP is likely to be more widely applied in future social development.

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