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A Simulation Method for Underground Water Flow Fields in Coal Mines Based on the Radial Basis Function Collocation Method

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Abstract. This paper uses radial basis functions to implement meshless modeling for the two-dimensional flow problem of groundwater in coal mines, and describes the modeling process. In practical applications, the radial basis function collocation method is used as the theoretical basis to describe the underground water flow field model of the Ordovician aquifer in Qianjiaying Mine. The model's scope is the hydrogeological unit where Qianjiaying Mine is located. A certain number of subdomains are divided in the model research area, and the actual hydraulic boundary of the flow field is taken as the model boundary. Then, a program framework is designed, and Python is used for program simulation. Finally, the simulation experiments prove that there is almost no significant difference between the exact solution and the approximate solution in the method described in this paper, and the graphs almost overlap. There is a very small absolute error between the exact solution and the approximate solution. The method described in this paper has much higher accuracy in the numerical simulation of two-dimensional steady-state flow and can better simulate actual problems.

Keywords: radial basis function, meshless modeling, two-dimensional flow field, numerical simulation

1 Introduction

Coal is a core component of China's energy structure and a major energy raw material supporting national economic construction and residents' lives. Given the characteristics of China's energy resource endowment and the consideration of national energy strategic security, even in the era of advocating for an optimized energy structure and green new energy, the energy structure with coal as the mainstay will not undergo substantial changes in the foreseeable future. Due to the complex geological and hydrogeological conditions of most coal seams in China's mining areas, water hazards from the roof and floor of coal mines have become one of the important factors threatening the safe production of coal mines. Therefore, on the basis of in-depth analysis of the hydrogeological conditions of the mining area and the mining field, constructing a groundwater flow field model of the coal mine and predicting the water inflow or drainage volume of the coal seam roof and floor is one of the key links in improving the three-dimensional prevention and control system and emergency rescue mechanism for coal mine water hazards, and is also an important technical means to ensure the safe production of coal mines [1].

The solution methods for the groundwater flow field model in coal mines are mainly divided into two categories: analytical methods and numerical methods. Among them, various flow field models and their solution methods in groundwater dynamics belong to the analytical method category; while numerical methods mainly include the finite difference method, the finite element method, etc. [2]. Although the analytical method has a relatively high solution accuracy, its idealized assumptions about the aquifer structure and flow field boundary conditions are difficult to fully conform to the actual water flow field morphology in the coal seam roof and floor aquifers. In contrast, numerical methods (such as the finite difference method and the finite element method) can better approximate the simulation of aquifer structure and flow field boundary conditions, but their solution accuracy is usually lower than that of the analytical solution, and adjusting the boundary conditions requires a large amount of work [3]. When solving the groundwater flow field problems of medium complexity, which account for the largest proportion in the coal mining process, the limitations of both the analytical method and the numerical

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method are particularly prominent, while their advantages are difficult to be fully exerted. In addition, there is an excessive reliance on groundwater numerical simulation software (such as Visual MODFLOW, FEFLOW, GMS, etc.) in the current research on the simulation of underground water flow fields in coal mines, which is specifically manifested in the lack of in-depth analysis of the hydrogeological conditions and hydrogeological test data of the mine, and only mechanical modeling based on a certain simulation software is carried out, and visual results can be obtained simply by inputting the original data.

In response to the above problems, this paper adopts the radial basis function collocation method to solve the common confined steady flow problem in the numerical simulation of groundwater flow, with a particular focus on the case where the permeability tensor is a piecewise constant [4].

The radial basis function (RBF) collocation meshless method has attracted much attention due to its directness, simplicity of principle, fast convergence speed, no need for numerical integration, and flexible node arrangement. This method has many advantages such as easy understanding, simple programming, high computational efficiency, and wide application range. It is particularly worth mentioning that, because the radial basis function has low sensitivity to spatial dimensions, this method has more advantages in solving high-dimensional problems compared with other numerical methods for solving differential equations.

The concept of radial basis functions can be traced back to 1971 when Hardy proposed the well-known multiquadric function and applied it to the research of irregular surface interpolation and terrain surface interpolation. In 1982, Franke tested 29 different algorithms in typical function interpolation problems and, based on standards such as time efficiency, accuracy and implementation difficulty, listed the multiquadric radial basis function and thin plate spline function as two better choices. Radial basis functions are composed of one-dimensional distance variables, have the characteristics of not depending on dimension and geometric complexity, and are easy to implemen.

The radial basis function method demonstrates excellent flexibility and convergence when addressing practical problems, especially in solving multi-dimensional problems, which is easier to implement compared to traditional methods. Therefore, it has become one of the important tools for numerically solving partial differential equations. The basic idea of the radial basis function collocation method is to arrange nodes within the research area, construct radial basis functions at these nodes, and substitute them into the problem equation. Then, through programming with computing software, the approximate solution of the equation can be obtained. As the first radial basis function collocation method, the Kansa method shows better convergence than traditional methods, and also has significant advantages such as integration and ease of implementation.

As a meshless method, the radial basis function collocation method has the global characteristic of its basis functions, which can flexibly deal with moving boundary problems, and its approximate solution has infinite smoothness. In addition, since this method is based on the Euclidean norm, it can be easily extended to the solution of n-dimensional numerical problems.

The development of open-source simulation methods for underground water flow fields based on radial basis functions is still in its infancy. It is necessary to select an appropriate programming language to describe the method proposed in this paper. The advantages of using Python for the development of open-source simulation programs for underground water flow fields in coal mines based on radial basis functions are as follows:

1) Using Python can achieve the design goal of “visible and modifiable at the bottom layer” for the open-source simulation program of underground water flow fields in coal mines based on radial basis functions. Programs written in Python have their source code publicly available. Other researchers in the same field can thoroughly understand the entire process of parameter input, model construction, model solution, and result output based on the source code of the flow field model program, thereby deepening their understanding of the theories and methods related to the model. In addition, other researchers can modify the program’s functionality from the bottom layer algorithm level by modifying the source code according to their research needs. This “visible and modifiable at the bottom layer” advantage is not possessed by flow field simulation programs developed using other non-open-source programming languages.
2) The use of the Subdomain Analytical Element Method Flow Field Model dramatically enhances the flexibility and accuracy of the simulation of underground water flow. By decomposing the computational domain into subdomains and using analytical solutions over each subdomain, Subdomain Analytical Element Method Flow Field Model provides more realistic representation of highly sophisticated geological and hydrogeological conditions. Subdomain Analytical Element Method Flow Field Model outperforms traditional numerical models by overcoming drawbacks related to having discontinuous hydrogeological parameters as well as heterogeneous geometries of aquifers. Application of Subdomain Analytical Element Method Flow Field Model with radial basis functions enables more flexible and cost-effective simulation of underground water flow, particularly where aquifer properties are spatially

different from one area of the coal mine to another.

3) Python is particularly suitable for scientific computing. The third-party modules NumPy and SciPy of Python play a very important role in the development of open-source simulation programs for underground water flow fields in coal mines based on the subdomain collocation method. The array operations and other functions in the NumPy module greatly facilitate the construction of the flow field model, while the linear equation solving sub-module in the SciPy module can significantly improve the computational efficiency of the flow field model.
4) Python, when combined with Jupyter Notebook and Matplotlib, can efficiently conduct interactive model running, full record of model operation, simulation result graphing, and analysis research. On the Jupyter Notebook platform, Python-written model programs can be interactively run and the entire model operation process can be fully recorded, ensuring the reproducibility of the model and facilitating other researchers to understand the details and steps of the model operation. The third-party module Matplotlib of Python is a simple and efficient program for graphing simulation results. It can generate various customized data graphs of simulation results based on the needs of analyzing and researching the flow field model by inputting relatively simple codes.
5) Python has extremely strong extensibility and expandability. The flow field simulation programs developed with Python can call the faster algorithm expansion modules written in C or C++ through the numerous APIs and tools provided by Python, thereby improving the operational efficiency of the entire simulation program. Additionally, the flow field model programs written in Python can also be called by C/C++ programs [5].

2 Related Work

There are relatively few studies on the construction of flow field models for groundwater in coal mines. Xiaolang Zhang from Ohio State University used HydroGeoSphere to build a two-dimensional cross-sectional numerical model of groundwater-surface water coupling, simulating the formation of an underground water flow system under the dynamic driving of the periodicity and randomness of rainfall. On the basis of collecting and organizing relevant hydrological and meteorological and survey data [6], Xingqu Lyu used the GMS (Groundwater Modeling System) numerical simulation software to establish an underground water flow model that conforms to the hydrogeological conditions of the study area, and predicted and analyzed the groundwater level and flow field in the study area under the influence of mining over the next 10 years, and analyzed the groundwater water balance state [7]. Jingdong Liu, with the help of FLAC3D numerical simulation software, analyzed the evolution law of the permeability of the floor during the mining process of the working face, quantitatively divided the changes in the permeability of the floor during the mining process, and clearly identified the instability of the permeability in different areas of the floor caused by coal mining. At the same time, using the GMS groundwater system simulation software, he analyzed the variation law of the groundwater level after the working face was mined under the conditions of no grouting and grouting of the floor. Based on data collection, dynamic long-term observation, and borehole pumping tests [8], Zexue Qi evaluated the natural resource quantity and allowable exploitation quantity of groundwater in the area by using the cross-sectional runoff method combined with numerical simulation at the basin scale, and assessed the impact of groundwater exploitation on the hydrological environment [9].

In the application of the radial basis function collocation method, considering the superiority of radial basis functions in fluid models, Hui Zhao proposed a new numerical simulation method for volumetrically fractured shale gas reservoirs. This method can effectively characterize the mass exchange between the matrix and fractures and achieve efficient and accurate simulation of the production process of volumetrically fractured shale gas reservoirs. The simulation results show that the numerical simulation method of volumetrically fractured shale gas reservoirs based on the collocation method significantly improves the computational efficiency and convergence while ensuring the calculation accuracy, especially for complex target solution problems, the improvement in computational efficiency is more significant [10].

Yu Wang from Shanghai Jiao Tong University constructed an unsteady explicit dynamic fluid-structure interaction analysis framework suitable for soft airships by using the radial basis function (RBF) and Delaunay mapping method. The reliability and accuracy of this numerical method were verified through the flat plate impact and NACA0014 wing aeroelasticity cases. The research shows that this computational framework is also applicable to the unsteady two-way fluid-structure interaction analysis of thin-film type aerostats such as high-altitude balloons. Finally, the structural response of a certain soft airship under different pressure difference conditions was reconstructed using the above framework [11].

Jianjian Xin proposed an immersed boundary method combined with the radial basis function virtual grid method for simulating viscous flow around complex or multi-body immersed boundaries. This method discretizes the incompressible Navier-Stokes equations on a fixed Cartesian staggered grid using the finite difference method and integrates in time with a third-order Runge-Kutta scheme. The convection terms are discretized using a high-order TVDMUSCL (total variation diminishing monotonic upstream-centered scheme for conservation laws) scheme. To account for the influence of sharp boundaries on the flow field, a continuous virtual grid method is introduced to apply the solid surface boundary conditions, and radial basis functions with polynomial bases are used to describe and reconstruct arbitrary complex immersed interfaces while identifying the background grid attribute states. Notably, this study was developed using Fortran90, and its development process can provide a reference framework for the simulation language development in this paper [12].

3 Establishment of Steady-State Water Flow Model

The description method of two-dimensional unsteady seepage of groundwater is as follows:

$$S\frac{\partial H}{\partial t}=\frac{\partial}{\partial x}(T\frac{\partial H}{\partial x})+\frac{\partial}{\partial y}(T\frac{\partial H}{\partial y})+f \text{ , } \mathbf{x}(x,y)\in\Omega,\ 0<t<T \quad \mathbf{(1)}$$

$$H(\mathbf{x},t)=g(\mathbf{x},t)\text{ , } \mathbf{x}\in\mathit{\Gamma}_1,\ 0<t<T \quad \mathbf{(2)}$$

$$T\frac{\partial H}{\partial \mathbf{n}}=q(\mathbf{x},t)\text{ , } \mathbf{x}\in\mathit{\Gamma}_2,\ 0<t<T \quad \mathbf{(3)}$$

$$H(\mathbf{x},0)=h_0(\mathbf{x})\text{ , } \mathbf{x}\in\Omega \quad \mathbf{(4)}$$

Here, $h(\mathbf{x},t)$ represents the head of water, $T(\mathbf{x})$ the coefficient of hydraulic conductivity, $S(\mathbf{x})$ the coefficient of storage, $f(\mathbf{x},t)$, $h_0(\mathbf{x})$ and $g(\mathbf{x},t)$ are given functions, Ω the seepage area, $\mathit{\Gamma}_1$ the first type of boundary of region Ω, and $\mathit{\Gamma}_2$ the second type of boundary of region Ω.

In the time interval $[0, T]$, the time node $0 = t^0 < t^1 < ... < t^m = T$ is set up. In the case of equal division, there is $t^n = t^{n-1} + \Delta t$. Let $Lh=\frac{\partial}{\partial x}(T\frac{\partial H}{\partial x})+\frac{\partial}{\partial y}(T\frac{\partial H}{\partial y})$, $H^n = H(\mathbf{x},t^n)$, $f^n = f(\mathbf{x},t^n)$, $g^n = g(\mathbf{x},t^n)$, $q^n = q(\mathbf{x},t^n)$, and for equation (1), the time variable t is discretized by the weight $\theta(0\le\theta<1)$. At $t = t^n$, there is:

$$S\frac{H^{n+1}-H^n}{\Delta t}\approx\theta(LH^{n+1}+f^{n+1})+(1-\theta)(LH^n+f^n) \quad \mathbf{(5)}$$

After sorting out, it can be obtained that:

$$SH^{n+1}-\Delta t\theta LH^{n+1}\approx SH^n+\Delta t(1-\theta)LH^n+\Delta t(1-\theta)f^n+\Delta t\theta f^{n+1} \quad \mathbf{(6)}$$

Approximate $h(\mathbf{x},t)$ with radial basis functions, that is:

$$H(\mathbf{x},t)\approx\sum_{j=1}^{N}\alpha_j(t)\varphi_j(\mathbf{x}) \quad \mathbf{(7)}$$

$$H(\mathbf{x},t^n)\approx\sum_{j=1}^{N}\alpha_j^n\varphi_j(\mathbf{x}) \quad \mathbf{(8)}$$

Here, when $\alpha_j^n = \alpha_j(t^n)$ holds, substituting formula (8) into formulas (6), (2), and (3) respectively and denoting the result as $F^n = \Delta t(1-\theta)f^n + \Delta t\theta f^{n+1}$, the following expressions are obtained:

$$\sum_{j=1}^{N}[S\varphi_j(\mathbf{x}) - \Delta t\theta L\varphi_j(\mathbf{x})]\alpha_j^1 = SH^0 + \Delta t(1-\theta)LH^0 + F^0 \text{ , } \mathbf{x} \in \Omega \tag{9}$$

$$\sum_{j=1}^{N}\alpha_j^1\varphi_j(\mathbf{x}) = g^1 \text{ , } \mathbf{x} \in \mathit{\Gamma}_1 \tag{10}$$

$$T\sum_{j=1}^{N}\frac{\partial\varphi_j(\mathbf{x})}{\partial\mathbf{n}}\alpha_j^1 = q^1 \text{ , } \mathbf{x} \in \mathit{\Gamma}_2 \tag{11}$$

$$\sum_{j=1}^{N}[S\varphi_j(\mathbf{x}) - \Delta t\theta L\varphi_j(\mathbf{x})]\alpha_j^{n+1} = \sum_{j=1}^{N}[S\varphi_j(\mathbf{x}) + \Delta t(1-\theta)L\varphi_j(\mathbf{x})]\alpha_j^n + F^n \text{ , } \mathbf{x} \in \Omega,\ n \geq 1 \tag{12}$$

$$\sum_{j=1}^{N}\alpha_j^{n+1}\varphi_j(\mathbf{x}) = g^{n+1} \text{ , } \mathbf{x} \in \mathit{\Gamma}_1,\ n \geq 1 \tag{13}$$

$$T\sum_{j=1}^{N}\frac{\partial\varphi_j(\mathbf{x})}{\partial\mathbf{n}}\alpha_j^{n+1} = q^{n+1} \text{ , } \mathbf{x} \in \mathit{\Gamma}_2,\ n \geq 1 \tag{14}$$

Nodes $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ are configured in region Ω and on its boundaries, where $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_{N_0}$ is an internal node in region Ω, $\mathbf{x}_{N_0+1}, \mathbf{x}_{N_0+2}, ..., \mathbf{x}_{N_0+N_1}$ is a first-class boundary node, $\mathbf{x}_{N_0+N_1+1}, \mathbf{x}_{N_0+N_1+2}, ..., \mathbf{x}_{N_0+N_1+N_2}$ is a second-class boundary node, N_0, N_1, N_2 is the number of internal nodes, first-class and second-class boundary nodes respectively, and the total number of nodes is $N = N_0 + N_1 + N_2$. Let $\varphi_{ij} = \varphi_j(\mathbf{x}_i)$, $L\varphi_{ij} = L\varphi_j(\mathbf{x}_i)$, $f_i^n = f(\mathbf{x}_i, t^n)$, $F_i^n = \Delta t(1-\theta)f_i^n + \Delta t\theta f_i^{n+1}$, $g_i^n = g(\mathbf{x}_i, t^n)$, $q_i^n = q(\mathbf{x}_i, t^n)$ be the result of substituting node $\mathbf{x}_i$ into formulas (9), (10), (11) and (12), (13), (14), respectively.

$$\sum_{j=1}^{N}[S\varphi_{ij} - \Delta t\theta L\varphi_{ij}]\alpha_j^1 = S\,H^0 + \Delta t(1-\theta)LH^0 + F_i^0 \text{ , } i = 1, 2, ..., N_0 \tag{15}$$

$$\sum_{j=1}^{N}\alpha_j^1\varphi_{ij} = g_i^1 \text{ , } i = N_0 + 1, N_0 + 2, ..., N_0 + N_1 \tag{16}$$

$$T\sum_{j=1}^{N}\left.\frac{\partial\varphi_j(\mathbf{x})}{\partial\mathbf{n}}\right|_{\mathbf{x}_i}\alpha_j^1 = q_i^1 \quad i = N_0 + N_1 + 1, N_0 + N_1 + 2, ..., N_0 + N_1 + N_2 \tag{17}$$

$$S\frac{H^{n+1} - H^n}{\Delta t} \approx \theta(LH^{n+1} + f^{n+1}) + (1-\theta)(LH^n + f^n) \tag{18}$$

$$\sum_{j=1}^{N}\alpha_j^{n+1}\varphi_{ij} = g_i^{n+1} \text{ , } i = N_0 + 1, N_0 + 2, ..., N_0 + N_1,\ n \geq 1 \tag{19}$$

$$T\sum_{j=1}^{N}\frac{\partial\varphi_j(\mathbf{x})}{\partial\mathbf{n}}\bigg|_{\mathbf{x}_i}\alpha_j^{n+1}=q_i^{n+1}\ ,\ i=N_0+N_1+1,N_0+N_1+2,...,N_0+N_1+N_2\ ,\ n\geq 1 \qquad \textbf{(20)}$$

First, solve for $\alpha_1^1,\alpha_2^1\cdots,\alpha_N^1$ from the linear equations determined by (15), (16), and (17), and then solve for $\alpha_1^{n+1},\alpha_2^{n+1}\cdots,\alpha_N^{n+1}$ layer by layer from the linear equations (18), (19), and (20). Substituting these into (8) yields the discrete approximate solution of the head function $h(x, y, t)$.

4 Framework Design for Underground Water Flow Field Simulation in Coal Mines

The method of constructing the framework structure of an open-source simulation program for underground water flow fields in coal mines using classes in Python is as follows. The instance corresponding to the class about pumping (injection) wells is the pumping (injection) wells in the flow field model. Applying the methods of the class about pumping (injection) wells to these wells can calculate the complex potential function value generated by the well and the flow between any two points in the flow field generated through the well. The instance corresponding to the class about line sources is the various external and internal boundaries (common boundaries between subdomains) of the flow field model. Applying the methods of the class about line sources to these flow field boundaries can subdivide the line sources on the simulated boundaries according to the simulation accuracy requirements of the model; can set the control points and normal flow control segments of the line sources on the simulated boundaries; can calculate the complex potential function value generated by the line sources on the simulated boundaries and the flow between any two points in the flow field generated through the line sources. The instance corresponding to the class about subdomains is each subdomain that makes up the flow field model. Applying the methods of the class about subdomains to these subdomains can generate the corresponding number of multi-quadratic radial basis function base points in each subdomain according to the simulation accuracy requirements of the flow field model. The instance corresponding to the class about the flow field model is the flow field model itself. Applying the methods of the class about the flow field model to the flow field model can set the various sub-time periods required for the simulation of the flow field model; can combine the relevant data of the instances of the class about subdomains, the class about line sources, and the class about pumping (injection) wells to solve the steady flow model or the unsteady flow model respectively.

The simulation of the inner and outer boundaries of the flow field is one of the core functions of the open-source simulation program for underground water flow in coal mines based on the radial basis function collocation method. The simulation program mainly realizes this function through four methods related to line sources: dividing the line sources according to the simulation accuracy requirements of the flow field model, setting control points and normal flow control segments for the line sources on the simulation flow field boundaries, calculating the complex potential function values in the flow field generated by the line sources, and calculating the flow between any two points in the flow field generated by the line sources. When simulating the inner and outer boundaries of the flow field using line source examples, the water levels of the line source control points or the normal flows of the normal flow control segments need to be set equal to the water levels or normal flows on the corresponding flow field boundary conditions. Applying the method of calculating the flow between any two points in the flow field generated by the line sources to the line source examples obtained after dividing according to the simulation accuracy requirements of the flow field model can yield expressions of the normal flow values that only contain the unknown parameters of the line source intensity. These expressions are components of the system of equations of the flow field model.

By comparing and analyzing the simulation results with those of the MODFLOW model and the FEFLOW model, the advantages of the radial basis function collocation method model in handling the groundwater flow field in coal mines are demonstrated [13]. The overall simulation framework process is shown in the Fig. 1.

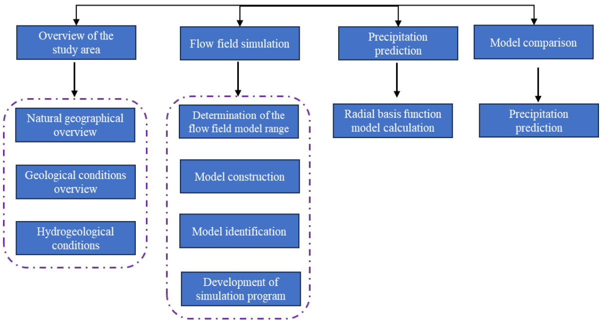

Fig. 1. Simulation framework flowchart

The target area is located in the southeast wing of the Kaiping Coalfield in Tangshan. The study area is characterized by well-developed fold structures, mainly distributed in the northeastern and southwestern parts of the study area, with a monocline structure in the middle. The fold curves within the mining field are clearly linearly arranged, mostly asymmetric, and the axes of each fold are oblique to the main syncline axis (Kaiping syncline). Faults are often associated with the fold axes and the transition zone between the fold and monocline areas. The coal-bearing strata in the Qianjiaying mining field are the Upper Carboniferous and Middle and Lower Permian strata, with the base being the Middle Ordovician Majiagou Formation limestone. The total thickness of the coal-bearing strata is approximately 420 meters. They are mainly composed of thick-bedded sandstone, claystone, limestone and coal seams. The coal-bearing strata contain more than twenty coal seams, with a total thickness of 18.94 meters. The main coal seams being mined are the 7th, 9th and 12-1st seams [14].

The actual location of the geological anomaly in Qianjiaying Coal Mine is the airway of the lower working face in the fourth mining area. The western part of the fourth mining area where the working face is located is the Nanyangzhuang anticline and its associated faults, and it is adjacent to the Fanggezhuang Mine in the east, with the eleventh mining area in the deep part. The structure is mainly composed of the Nanyangzhuang anticline and the Bigezhuang syncline, and the associated faults and secondary small synclines form a relatively complex structural pattern in this area. The 5th coal seam in the hydrogeological region 1 and 2 is unmineable, and its roof aquifer has locally abundant water, with a water head height estimated at about 100 meters, which provides a certain amount of water recharge to the roof of the 7th coal seam. Based on the analysis of the water volume changes in the working face, it all belongs to the normal water outflow from the roof of the coal seam. Without new mining activities, the water volume shows a decreasing trend, which is consistent with the water outflow characteristics of the 7th coal seam in other areas. The roof of the 9th coal seam is a weak aquifer, and the water volume of the working face is generally small. There is no obvious water outflow in the tunneling and mining working faces. The maximum expected water inflow in the tunneling working face is 0.1 cubic meters per minute, and the normal water inflow is 0.05 cubic meters per minute. During the tunneling of the airway and transport way, there is only a small amount of dripping water in some local areas. The sandstone fissure aquifer above the 12-1 coal seam has weak water-bearing capacity due to the drainage caused by the exposure of the roadway. The sandstone fissure aquifer below the 12-1 coal seam has locally strong water-bearing capacity, but there is no obvious water outflow during the construction of the working face. Geological exploration has been carried out in the past in areas 1 and 3, with an exploration area of 2.59 square kilometers. A total of 14 faults were interpreted. Except for a reverse fault about 150 meters ahead of the face (where the geological anomaly was actually seen), the 3D seismic data shows no other anomalies in the airway.

4.1 Simulation of Groundwater Flow Field in Coal Mines

The radial basis function collocation method groundwater flow field model of the Ordovician aquifer in Qianjiaying Mine is based on the radial basis function collocation method described in the previous text. The model's scope is the hydrogeological unit where Qianjiaying Mine is located. A certain number of subdomains are divided in the model research area, and the actual hydraulic boundary of the flow field is used as the model's boundary. The boundary of the model is divided into equidistant straight line segments and simulated by the nonlinear intensity high-order line source complex potential function created in the previous text. The flow field within each subdomain divided by the model is simulated by the well flow complex potential function described in the previous text. The model uses the regional sink function represented by multiple quadratic radial basis functions to simulate unsteady flow.

Based on the data from borehole exploration and pumping tests, the karst development zone of the Ordovician limestone in Qianjiaying Mine is mainly concentrated within a vertical range of 95 meters from the top of the Ordovician limestone. The non-karst development zone of the limestone 95 meters below the top of the Ordovician limestone and the siltstone and mudstone at the bottom of the Jurassic system respectively serve as the bottom and top impermeable layers of the Ordovician limestone aquifer, isolating the hydraulic connection between this aquifer and adjacent aquifers. Therefore, the Ordovician limestone aquifer in the flow field model is generalized as a single aquifer. The constant head boundary, normal flow boundary, and impermeable boundary in the flow field model are all simulated using the fourth-order line sinks with intensity nonlinear variation as described in the previous text. Considering the balance between the simulation accuracy of the model and the computational efficiency, the line sinks simulating various hydraulic boundaries in the model are divided into several straight-line segments. The division length of the line sinks is set at 200 meters, meaning that each divided line sink is 200 meters or less in length. Four equally spaced line sink control points and four normal flow control segments are set on each line sink (the four normal flow control segments are determined by the endpoints of the line sink and the midpoints of the straight-line segments formed by adjacent line sink control points, to meet the control conditions of water level or normal flow for the corresponding hydraulic boundaries).

The unsteady flow in the flow field model is simulated by using the zone sink represented by the multi-quadratic radial basis function discussed in the previous text. The base points of the zone sink are uniformly distributed within the study area. Considering the computational cost and simulation accuracy of the model, the distance between adjacent base points in this model is set to 300m. The growth rate of the time step for the unsteady flow is set to 1.5, that is, the time step of the next period is 1.5 times that of the previous period.

4.2 Simulation Program Design

The specific objects created based on the attributes and methods defined in a class are called instances. Classes play a very important role in the framework design of the open-source simulation program for underground water flow fields in coal mines based on the radial basis function collocation method. The program design framework is shown in Fig. 2.

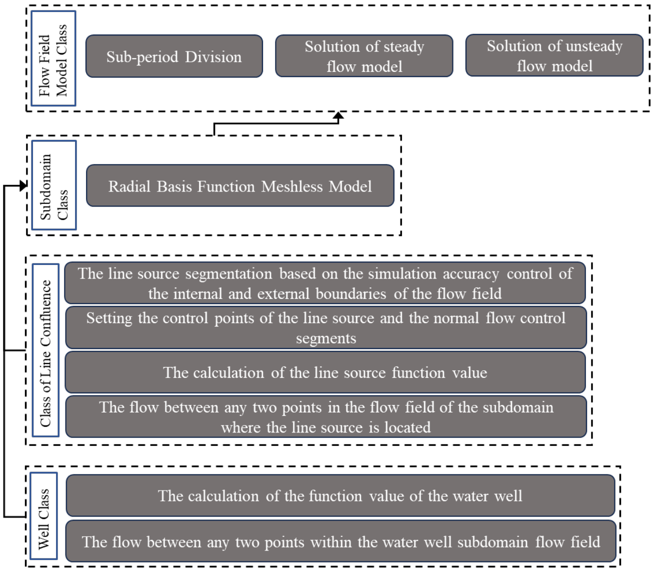

Fig. 2. Schematic diagram of the simulation program framework

Starting from this section, the design of the development framework for an open-source simulation program of underground water flow fields in coal mines based on the subdomain element method will be carried out using the specific Python language and third-party Python modules. This section will apply specific code to elaborate on the development content of the simulation program from aspects such as subdomain setting in the simulation program, simulation of internal and external boundaries of the flow field, simulation of pumping (injection) wells, simulation of unsteady flow, control of simulation accuracy of the flow field model, generation of the system of equations of the model, and application of third-party modules and Jupyter Notebook. In the simulation program, a class named Subdomain is created for subdomains, and the __init__ method is applied to initialize the class Subdomain. The main functions of this method include assigning hydrogeological parameters of the aquifer in the subdomain (thickness of the aquifer, permeability coefficient, storage (release) coefficient, and initial water level), specifying the distance between the base points of the unsteady flow area, specifying the flow field model to which the subdomain belongs, setting the list (List) for collecting line sources and boundary line sources within the subdomain, setting the list for collecting the vertices of the subdomain polygon, setting the list for collecting base points within the subdomain, and setting the list for collecting pumping (injection) wells within the subdomain, etc. The definitions of each function and array list are shown in Table 1.

Table 1. Parameter description list of application class definition

Definitions of functions and various classes	Explanation of definitions
class Subdomain:#	Define the class Subdomain regarding subdomains.
def _init_(self,ml,H,K,Ss,M,ho): #	H, K, Ss, h represent the thickness, permeability coefficient, storage (release) coefficient, and initial water level of the Subdomain# sub-aquifer respectively, and M represents the Subdomain# sub-aquifer.
self.elementList,self.elementList2=[],[]	Collect line sources within subdomains using a list
self.elementtype,self.elementtype2=[],[]	Collect the flow field boundaries by classification with a list.
self.H=H self.K=Kself.ho=hoself.Ss=Ssself.M=M self.S=self.Ss*self.H	
self.model.subdomainList.append(self)	Add the subdomain to the subdomain list of the flow field model
self.vertex=[] sclf.vertex2=1	For placing the vertices of subdomain polygons
self.stwellList=[1]	Stable production (injection) wells with a fixed flow rate for placement within subdomains
self.trwellList=[]	Fixed-flow non-steady pumping (injection) wells for placement within subdomains
self.basepointList=[] self.ipoi=[] self.ipoihead=[] self.tripoihead=[]self.dh=[] self.bounds=[]	The base point for the non-stable flow area within the subdomain is used for placement.

The system of equations for constructing the flow field model is the most crucial part in the development of an open-source simulation program for underground water flow in coal mines based on the subdomain analytical element method. Only when the system of equations is correctly established can the entire flow field model be ultimately solved. The model system of equations is divided into two categories: the system of equations for the steady flow model and the system of equations for the unsteady flow model. The former is constructed by applying the solve1( ) method of the Model class, while the latter is constructed by applying the solve2( ) method of the Model class. The number of equations in the obtained model system of equations is equal to the number of unknown parameters in the flow field model, and the system of equations is a linear equation system. Therefore, the steady flow field model can be solved. In the unsteady flow model, the number of unknown parameters related to the line sinks on the inner and outer boundaries of the simulated flow field and the construction process of the equations for this type of unknown parameters are the same as those in the steady flow field model. Different from the steady flow model, the unsteady flow model also contains unknown parameters related to the regional sink base points represented by multiple quadratic radial basis functions for simulating unsteady flow. According to the principle and method of simulating unsteady flow with multiple quadratic radial basis function regional sinks as described in the previous text, the number of this type of unknown parameters is equal to the sum of the number of base points in the list of each subdomain and the number of subdomains [15].

The pseudo-code for the simulation process is as follows:

```
  def __init__(
          self,
          x_start: float,
          y_start: float,
          x_end: float,
          y_end: float,
          left_domain: Union[Dict, None] = None,
          right_domain: Union[Dict, None] = None,
          boundary_type: str = “internal”,
          control_points: Optional[List[Tuple[float, float]]] = None,
          normal_flux_segments: Optional[List[Tuple[float, float]]] = None
  # Set the geometric properties of the line sink
          self.x_start = x_start
          self.y_start = y_start
          self.x_end = x_end
          self.y_end = y_end
  # Store the adjacent domains or boundary conditions
          self.left_domain = left_domain
          self.right_domain = right_domain
  # Classify the boundary type
          self.boundary_type = boundary_type.lower()
          self._validate_boundary_type()
  # Add starting point to the left domain’s vertex list (if internal boundary)
          if self.boundary_type == “internal” and left_domain is not None:
              if hasattr(left_domain, ‘vertices’):
                  left_domain.vertices.append((x_start, y_start))
  # Initialize control points (for potential calculations)
          self.control_points = control_points if control_points is not None else []
  # Initialize normal flux segments (for flux-specified boundaries)
           self.normal_flux_segments = normal_flux_segments if normal_flux_segments is
not None else []
  # Calculate derived geometric properties
          self.length = self._calculate_length()
          self.angle = self._calculate_angle()
  def _validate_boundary_type(self):
          “““Validate that the specified boundary type is supported.”””
            valid_types = {“internal”, “external”, “head_specified”, “flux_specified”,
“no_flow”}
          if self.boundary_type not in valid_types:
               raise ValueError(f”Invalid boundary type: {self.boundary_type}. Must
be one of {valid_types}”)
  def _calculate_length(self) -> float:
          “““Calculate the length of the line sink segment.”””
             return ((self.x_end - self.x_start)**2 + (self.y_end - self.y_
start)**2)**0.5
  def _calculate_angle(self) -> float:
            “““Calculate the angle (in radians) of the line sink relative to x-ax-
is.”””
          dx = self.x_end - self.x_start
          dy = self.y_end - self.y_start
          return np.arctan2(dy, dx)
   def __repr__(self) -> str:
          “““String representation of the Linesink2 object.”””
          return (f“Linesink2(start=({self.x_start}, {self.y_start}), ”
                  f“end=({self.x_end}, {self.y_end}), ”
                  f“type={self.boundary_type})”)
```

5 Simulation Results and Analysis

The simulation results of the subdomain analytical element method model, MODFLOW model and FEFLOW model are compared and analyzed from the aspects of Theis well flow simulation and the prediction of the amount of water to be drained from the Ordovician aquifer due to coal mining in the previous section, in order to verify the simulation accuracy and computational efficiency of the subdomain analytical element method flow field model. The flow field models involved in the underground water drainage test and the prediction of the amount of water to be drained in the previous example of this section are all single-layer confined aquifer unsteady subdomain analytical element method flow field models generated by pumping (draining) wells. Therefore, the Theis well flow model with an analytical solution can fully test the simulation accuracy of the subdomain analytical element method flow field model in simulating this type of flow field. The water level values of the same Theis well flow (initial water level is 880m, well flow rate is 80m3/d, transmissivity is 90m2/d, storage coefficient is 10000) are calculated respectively by using the Theis formula, MODFLOW model and subdomain analytical element method model. The distance between adjacent nodes of the MODFLOW model is 1.45m to 4.99m, with an average distance of 3.79m. The distance between adjacent nodes of the FEFLOW model is 1.45m to 4.99m, with an average distance of 3.79m. The distance between the multiple quadratic radial basis function convergence points of the subdomain analytical element method model is 4.26m. The grid cell sizes of the three models are basically the same. The water level calculation results of the Theis well flow of the MODFLOW model and the subdomain analytical element method model are compared and analyzed. The comparison results are shown in Fig. 3. The formulas for calculating the absolute deviation and relative deviation of the water level calculation values of the models involved in the comparison analysis in this section are expressed as:

$$d_A = v_{cal} - s_{exa} \tag{21}$$

$$d_R = \frac{d_A}{s_{exa}} \times 100\% \tag{22}$$

In the formula, d_A represents the absolute deviation of the model water level calculation value, v_{cal} represents the model water level calculation value, s_{exa} represents the exact solution of the water level, and d_R represents the relative deviation of the model water level calculation value.

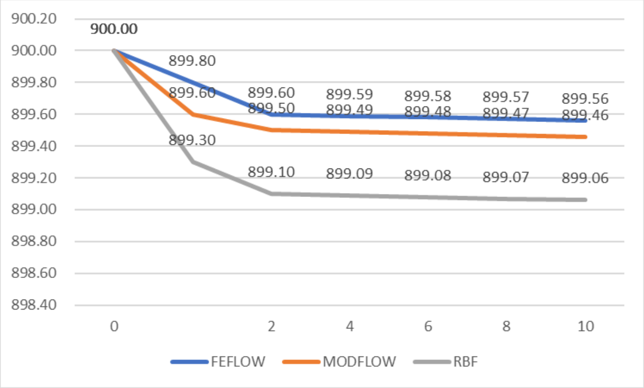

Fig. 3. The comparison chart of water level calculation results of Theis well flow

This section calculates the Theis well water level values at three positions 20 meters away from the well center in the flow field over a period of ten days using the MODFLOW model and the subdomain analytical element method model. It can be seen that the Theis well water level values calculated by the subdomain analytical element method model at this 20-meter position are closer to the exact solution of the Theis well water level than those calculated by the MODFLOW model. As shown in the figure, the fitting degree of the Theis well water level values calculated by the subdomain analytical element method model to the exact solution of the Theis well water level is higher than that of the MODFLOW model. Table 2 statistically analyzes the deviations of 400 sets of Theis well water level values calculated by the MODFLOW model and the subdomain analytical element method model from the corresponding exact values of the Theis well water level. The statistical results show that the absolute deviation range of the Theis well water level values calculated by the subdomain analytical element method model from the exact solution of the Theis well water level is -0.11m to 0.05m, while that of the MODFLOW model is -0.14m to 0.29m. The proportion of data with an absolute value of the relative deviation of the Theis well water level values calculated by the subdomain analytical element method model from the exact solution of the Theis well water level less than 0.05% is 98.76%, while that of the MODFLOW model is 19.31% respectively. Therefore, the deviation of the Theis well water level values calculated by the subdomain analytical element method model is significantly smaller than that of the MODFLOW model.

Table 2. Subdomain-analytic element method model and the MODFLOW model

	Subdomain analytical element method model	MODFLOW model
Absolute deviation (m) Relative deviation	200	200
The proportion of data with an absolute relative deviation less than 0.01%	-0.05m to -0.11m	-0.14m to 0.29m
The proportion of data with an absolute relative deviation less than 0.02%	71.32%	16.29%
The proportion of data with an absolute relative deviation less than 0.05%	81.07%	19.31%
Absolute deviation (m) Relative deviation	98.76%	52.48%

6 Conclusion

This paper mainly aims to explore the feasibility of using the improved radial basis function to model the underground water flow in coal mines for the simulation of the underground water flow field. Firstly, the complex potential function of the high-order line source with nonlinearly varying intensity (the intensity expression is a high-order standard polynomial) was created, and the spatial distribution laws of the complex potential, flow potential, and stream function values of this type of line source in the flow field were strictly demonstrated by mathematical methods. Then, the created high-order line source with nonlinear intensity was applied to simulate the inner and outer boundaries of the flow field, and the multi-quadratic radial basis function represented zone source was used to simulate the unsteady flow in the flow field; a Python-based open-source simulation program for the underground water flow field in coal mines was developed based on the subdomain analytical element method. Finally, this model and simulation program were applied to simulate the water flow field of the Ordovician limestone aquifer in Qianjiaying Mine. Through the comparative analysis with the simulation results of the MODFLOW model, the advantages of the subdomain analytical element method model in simulating the underground water flow field in coal mines were demonstrated.

The simulation of the underground water flow field in coal mines is a very complex dynamic process. It is necessary to restore the details of the flow field to the greatest extent and, in accordance with the requirements of the “Detailed Rules for the Prevention and Control of Water in Coal Mines”, through simulation, to specifically identify the hydraulic characteristics of the water-bearing aquifer flow field that threaten the safe mining of coal seams, thereby providing quantified flow field information for the formulation of water prevention and control plans in coal mines and pointing out the direction for possible subsequent hydrogeological exploration and detailed and complex simulations. The research in this paper shows that the radial basis function is a simulation method suitable for moderately complex underground water flow field problems in coal mines, with advantages such as fast modeling speed, flexible model adjustment, high solution accuracy, and high computational efficiency. However, there are still several aspects that need in-depth research for improvement and refinement:

(1) As most of the hydraulic element complex potential functions in this method are analytic functions on the complex plane, they are naturally suitable for simulating two-dimensional flow in a plane. Currently, most models are two-dimensional flow field models, and it is necessary to explore the construction of new element functions to simulate three-dimensional flow.

(2) The hydrogeological parameters of the aquifer (such as permeability coefficient, storage coefficient, etc.) and other aquifer attributes (such as aquifer layer number, aquifer bottom elevation, aquifer thickness, etc.) in different subdomains are discontinuously changing. Exploring the establishment of a flow model with continuously changing aquifer parameters and other attributes between different subdomains can more objectively simulate the flow field state of the study area.

(3) The flow potential functions of each hydraulic element in the flow field are superimposed to solve, so the sparsity of the coefficient matrix of the model system equation group is less than that of the coefficient matrix of the discrete model equation group such as finite difference and finite element. Moreover, as the number of hydraulic elements in the flow field increases and the simulation accuracy of the model is raised, the sparsity of the coefficient matrix of the model system equation group will become smaller and smaller. In the future, further in-depth research is needed in the optimization of algorithms for solving the model system equation group. The combination of the radial basis function collocation method and the subdomain analytical element method enhances the precision and flexibility of underground water flow simulation by facilitating the accurate handling of nonuniform hydrogeologic conditions and providing an efficient hybrid model for better analysis of unsteady flow and hydraulic properties affecting mining safety.

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