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A SHAP-Based Nonlinear Analysis Approach to Audit Identification of Patient Capital

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Abstract. Patient capital, with its strong financial capacity and risk prevention advantages, safeguards high-risk innovative enterprises and also influences auditors' professional judgment. Using a sample of Shanghai and Shenzhen A-share listed companies from 2011 to 2024, this paper investigates whether auditors can recognize the risk prevention effects of patient capital and thereby reduce audit pricing. The findings reveal that patient capital significantly reduces audit fees. Mechanism tests show that patient capital lowers audit pricing by reducing auditors' business risk and audit risk. Heterogeneity analysis indicates that the inhibitory effect of patient capital on audit pricing is more pronounced in firms with low ownership concentration, high market competition, and those undergoing digital transformation. Further SHAP-based nonlinear analysis demonstrates that auditors' risk perceptions of the two types of patient capital differ fundamentally: debt capital exerts governance effects only within a certain leverage range, and pushes pricing upward beyond the threshold, exhibiting a "threshold response" pattern; equity capital exhibits diminishing marginal effects as scale expands, displaying a "safety cushion" pattern. This study enriches the research on the economic consequences of patient capital investment and the determinants of audit pricing, providing a reference for auditors to identify the advantages of patient capital.

Keywords: patient capital, audit pricing, business risk, audit risk

1 Introduction

Against the backdrop of increasing global economic uncertainty and accelerating technological iteration, enterprises face severe challenges in long-term value creation and sustainable development. Patient capital, as a special capital form that combines long-term investment horizons, risk tolerance, and social value integration capabilities, nourishes the development of technological innovation and the cultivation of new quality productive forces through substantial capital flows. In April 2024, the Politburo of the CPC Central Committee first proposed to "actively develop venture capital and expand patient capital." In March 2025, "patient capital" was formally incorporated into the Government Work Report, marking its strategic upgrading from departmental initiatives to national top-level design. By October 2025, Zhejiang and Jiangsu provinces successively launched social security science and innovation funds, signaling the deep deployment of "national-level" patient capital in the science and innovation sector. Notably, patient capital typically flows into areas such as technological innovation, strategic emerging industries, and core key technologies, which are characterized by long payback periods and relatively high risks. As important stakeholders of enterprises, auditors pay particular attention to the inherent risks of industries. According to the audit pricing model [1], audit fees consist of auditors' cost inputs, risk premiums, and reasonable profits. Typically, auditors devote more audit resources to high-risk areas, leading to higher audit costs, while also charging higher risk premiums to compensate for potential audit risks. However, patient capital is characterized by strong risk-bearing capacity and risk prevention advantages [2]. Thus, whether high-risk enterprises receiving patient capital inputs also experience increased risk premiums, and whether auditors can recognize the risk prevention effects of patient capital and subsequently reduce audit fees, become topics worthy of exploration.

Audit pricing has long been an important topic of concern in both audit theory and practice. Firms' business model transformations [3] and the construction of financial shared service centers [4] are accompanied by high

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risks and significant uncertainty, yet research shows that both can significantly reduce audit risk premiums and thereby lower audit fees. Meanwhile, existing studies have summarized the determinants of audit pricing from three perspectives: accounting firms [5], clients [6], and other stakeholders affecting clients [7]. However, most studies have overlooked the possibility that audit clients' investors, as important stakeholders, may influence audit pricing. Therefore, this paper takes the impact of patient capital investors on firms as the entry point to explore whether and how patient capital affects audit pricing.

Although the term "patient capital" was first mentioned in the Politburo meeting, its investment practice has long existed, formally emerging in the United States in the 1980s [8]. In recent years, existing literature on the economic consequences of patient capital has examined new quality productive forces [9-10], innovation activities [11-13] , and corporate governance. However, few scholars have explored the impact of patient capital on corporate stakeholders. Theoretically, patient capital possesses an inherent mechanism for reducing audit pricing. The infusion of patient capital effectively prevents and controls internal and external risks for enterprises, alleviates operational risks and financial fraud risks, and helps reduce the business risk and audit risk faced by auditors, thereby lowering audit pricing. Whether this holds true in practice has yet to be empirically verified. Accordingly, this paper employs a sample of Shanghai and Shenzhen A-share listed companies from 2011 to 2024 to empirically examine the impact of patient capital on audit pricing and its underlying mechanisms.

The potential marginal contributions of this paper are twofold. First, from the auditor's perspective, it extends the literature on the economic consequences of patient capital. Distinct from existing studies that explain audit pricing differentials from the perspective of audit input costs, this paper centers its analytical focus on auditors' risk perception mechanisms, revealing that patient capital influences audit pricing through "risk signaling transmission" rather than "cost structure optimization." This finding extends the determinants of risk premium within Simunic's (1980) audit pricing framework, elevating shareholder structure characteristics from "peripheral variables" to "core driving factors" in audit pricing models.Existing studies have explored new quality productive forces, innovation activities, and corporate governance, with few focusing on whether patient capital affects stakeholder decision-making. This paper, from the perspective of auditors' pricing decisions, confirms the advantages of patient capital in preventing and controlling operational risks and financial fraud risks, thereby reducing auditors' business risk and audit risk and transmitting to lower audit pricing. Moreover, the impact of patient capital on audit pricing varies with ownership concentration, market competition, and digital transformation. Second, from the perspective of audit clients' investors, it extends the literature on determinants of audit pricing. Existing studies have examined audit pricing determinants from the angles of accounting firms, clients, and other stakeholders, while few have explored the economic consequences of "patient" investment and its impact on audit pricing. This paper empirically demonstrates that seemingly "high-risk" firm characteristics do not necessarily lead to high audit risk premiums, and that investors' risk prevention can effectively reduce firms' operational and financial risks, thereby lowering auditors' risk premiums and audit pricing.

2 Theoretical Analysis and Research Hypothesis

When entering into audit engagements with audited entities, auditors consider clients' operational risk, auditors' business risk, and audit risk as the most important factors, and clients' operational risk is closely related to auditors' business risk [14]. Although patient capital typically flows into high-risk areas, it generates positive effects across multiple dimensions including goal orientation, risk resolution, and governance participation, effectively preventing and controlling internal and external risks and reducing auditors' business risk and audit risk, thereby lowering audit pricing.

On the one hand, patient capital can mitigate operational risk at the firm level, thereby reducing auditors' business risk and subsequently lowering audit pricing. First, patient capital can alleviate firms' financing constraints and enhance their going-concern capabilities. Patient capital not only provides long-term capital support but also offers high-risk tolerance and strategic resource support, enabling firms to survive adversity and maintain sustainable development momentum. From a long-term investment perspective, while addressing firms' financing difficulties, patient capital places greater emphasis on the sustainability of enterprise development. Second, patient capital investors actively participate in corporate governance to mitigate idiosyncratic risks. In practice, patient capital can be categorized into strategic equity and relational debt, each with distinct mechanisms of influence on firms. Strategic equity typically comes from institutional investors such as pension funds and insurance companies. These investors generally participate in corporate governance as moderate monitors to ensure the maximization of firms' long-term interests. Relational debt, by contrast, generally takes the form of

long-term, illiquid debt instruments such as bank long-term loans and bonds. The providers of such funds may include banks that have established long-term relationships with firms, or financial institutions with long-standing cooperative relationships with specific enterprises. Relational debt participates in corporate governance through various investment and financing advisory services, helping firms optimize capital structure and providing advice on specific projects or strategies. Although the industries receiving patient capital investments are characterized by high uncertainty and risk, if auditors can recognize the risk-sharing conditions of firms in these industries, their risk assessments will correspondingly decrease, leading to lower risk premiums and reduced audit fees. Third, patient capital is characterized by stability and can withstand economic cycle shocks. Unlike short-term capital that overreacts to market fluctuations, patient capital internalizes economic cycle fluctuations as an organic component of investment strategy. When short-term market volatility occurs, such capital instead increases investment efforts, can withstand short-term market fluctuations without withdrawal, and exhibits high loyalty and stability. Adhering to value investing principles, patient capital not only establishes close financial ties and business cooperation with firms but also demonstrates positive effects in enhancing stock pricing efficiency and stock price informativeness, thereby improving firms' operating conditions through efficient financial resource allocation. In summary, patient capital possesses the advantages of sustainability, risk sharing, and stability, effectively mitigating operational risk at the firm level. Consequently, auditors face relatively lower business risk, leading to reduced audit pricing.

On the other hand, patient capital can effectively curb corporate financial fraud, reduce the level of material misstatement risk in financial statements, and thereby lower audit risk and audit pricing. First, patient capital investors, as external governance entities, play an important role in external monitoring and governance. Based on long-term oriented investment philosophies and specialized information analysis capabilities, patient capital possesses unique advantages in information acquisition and processing. Since patient capital investors' interests are aligned with those of the firm, they have strong incentives to discover and transmit internal firm information. In this process, they effectively improve the quality of corporate accounting information disclosure, alleviating information asymmetry while also helping to ease firms' financing constraints. Second, patient capital investors enhance firms' internal governance and improve accounting conservatism and comparability. Accounting conservatism is an effective manifestation of the prudence principle in accounting information quality requirements, which stipulates that the recognition of good news similar to gains should be subject to more stringent verification than the recognition of bad news similar to losses. Compared with retail investors, institutional investors typically participate directly in corporate governance as shareholders or external board members, playing an active role in firms' daily operations. They have both the capability and the opportunity to monitor executives' behavior, improve accounting information transparency and comparability, and reduce earnings management activities. Moreover, the higher their shareholding ratio, the stronger the contemporaneous accounting conservatism [15]. Third, patient capital helps alleviate managerial myopia and reduce financial fraud motives. According to the fraud triangle theory, fraudulent behavior occurs in the presence of pressure, opportunity, and rationalization. Based on principal-agent theory, due to the conflict of interest between managers and shareholders, when managers overemphasize quarterly earnings, this not only induces managerial myopia and neglect of long-term development but also increases managers' pressure to engage in financial statement manipulation, seek opportunities to obtain performance-based rewards, and rationalize such fraudulent behavior. Patient capital, however, places greater emphasis on firms' intrinsic values such as technological barriers, team capabilities, and industrial ecosystem positioning, and is insensitive to short-term financial data fluctuations. By extending the investment return period and proactively reducing the weight of short-term financial indicators in performance evaluation, patient capital can alleviate such incentive misalignment and effectively reduce managers' short-termism. Therefore, patient capital, through external monitoring, internal governance, and alleviation of managers' short-term pressure, curbs the possibility of financial fraud, improves financial information quality and transparency, reduces auditors' assessment of material misstatement risk in financial statements, and lowers audit risk, thereby reducing audit pricing.

In summary, this paper proposes the following research hypothesis:

Hypothesis H1: Ceteris paribus, patient capital can reduce audit pricing.

To further clarify the sufficient conditions under which patient capital reduces audit pricing, we construct a parsimonious model based on the audit pricing framework of Simunic (1980). Let audit pricing (*FEE*) be composed of audit costs (Cost), risk premium (*RiskPremium*), and normal profit (π):

$$FEE = \gamma_0 + \gamma_1 \cdot Cost + \gamma_2 \cdot RiskPremium + \pi \quad (1)$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$. The risk premium is jointly determined by the business risk (*BRISK*) and audit risk

(*ARISK*) faced by auditors, i.e.:

$$\begin{cases} RiskPremium = h\left(BRISK, ARISK\right) \\ \partial h / \partial BRISK > 0 \,\&\, \&\partial h / \partial ARISK > 0 \end{cases} \tag{2}$$

Let the amount of patient capital investment be P (where P refers to either strategic equity *EQUITY* or relational debt *DEBT*). Based on the preceding theoretical analysis, patient capital exerts suppressive effects on firms' operational risk and financial fraud risk through external monitoring, governance participation, and long-term financial support, which can be expressed as:

$$\begin{cases} BRISK\left(P\right) = b_0 - b_1 \cdot P + u \quad b_1 > 0 \\ ARISK\left(P\right) = a_0 - a_1 \cdot P + v \quad a_1 > 0 \end{cases} \tag{3}$$

Where u and v are random disturbance terms, and b_1 and a_1 measure the marginal mitigating effects of patient capital on business risk and audit risk, respectively. Substituting these two equations into the audit pricing equation and taking the partial derivative with respect to P:

$$\begin{aligned} \partial FEE / \partial P &= \gamma_2 \cdot \begin{bmatrix} \left(\partial h / \partial BRISK\right) \cdot \left(\partial BRISK / \partial P\right) + \\ \left(\partial h / \partial ARISK\right) \cdot \left(\partial ARISK / \partial P\right) \end{bmatrix} \\ &= \gamma_2 \cdot \left[- \left(\partial h / \partial BRISK\right) \cdot b_1 - \left(\partial h / \partial ARISK\right) \cdot a_1 \right] < 0 \end{aligned} \tag{4}$$

Since $\gamma_2 > 0$, $\partial h / \partial BRISK > 0$, $\partial h / \partial ARISK > 0$, and $b_1 > 0$, $a_1 > 0$, it follows that $\partial FEE / \partial P < 0$ holds unconditionally. This derivation theoretically establishes that as long as patient capital effectively reduces firms' operational risk (i.e., $b_1 > 0$ and audit risk (i.e., $a_1 > 0$), its inhibitory effect on audit pricing is logically guaranteed, providing a solid mathematical foundation for the subsequent empirical analysis.

3 Research Design

3.1 Sample and Data Sources

This paper takes Shanghai and Shenzhen A-share listed companies from 2011 to 2024 as the initial sample. After excluding financial firms, firms with abnormal operations such as ST or ST* designations, and observations with missing data, we obtain 24,219 firm-year observations. The data are sourced from the China Stock Market and Accounting Research (CSMAR) database. All continuous variables are winsorized at the 1st and 99th percentiles.

3.2 Variable Definitions

Patient Capital. Following the measurement approach of Jiang and Wu [16], this paper measures firms' patient capital ratio from two dimensions: strategic equity and relational debt. For strategic equity, we use the shareholding ratio of strategic institutional investors. Specifically, we calculate the ratio of institutional investors' shareholding to the standard deviation of institutional investors' shareholding over the past three years. If a firm's ratio in a given year is not lower than the industry-year median, the investor is classified as a strategic institutional investor; otherwise, it is classified as a transactional institutional investor [17]. For relational debt, we use the ratio of listed companies' long-term debt to total liabilities.

Audit Pricing. Following prior studies, this paper measures audit pricing as the natural logarithm of the total audit fees paid by the firm in the current year. A higher value indicates higher audit pricing.

Control Variables. Following prior studies, this paper controls for a series of factors that influence audit pricing, including firm size, leverage ratio, profitability, return on total assets, firm growth, board size, proportion of independent directors, CEO duality, largest shareholder shareholding ratio, firm age, and auditor opinion. The paper also controls for firm and year fixed effects, with standard errors clustered at the firm level. Detailed variable definitions are presented in Table 1.

Table 1. Variable definitions

Variable type	Variable name	Symbol	Definition
Dependent variable	Audit pricing	FEE	Natural logarithm of total audit fees paid by the firm in the current year
Independent variables	Patient capital	EQUITY	Shareholding ratio of strategic institutional investors
		DEBT	Long-term debt / Total liabilities
Control variables	Firm size	SIZE	Natural logarithm of total assets
	Leverage	LEV	Total liabilities / Total assets
	Return on total assets	ROA	Net income / Total assets
	Firm growth	GROWTH	Annual revenue growth rate
	Board size	BOARD	Natural logarithm of the number of board directors
	Independent director ratio	INDEP	Number of independent directors / Number of board directors
	CEO duality	DUAL	Dummy variable equal to 1 if the chairman and CEO are the same person, and 0 otherwise
	Largest shareholder shareholding	TOP1	Number of shares held by the largest shareholder / Total shares outstanding
	Firm age	AGE	Natural logarithm of (number of years since firm establishment + 1)
	Auditor opinion	AUDOPIN	Dummy variable equal to 1 if a standard unqualified opinion is issued, and 0 otherwise

3.3 Model Specification

To test our hypothesis, we construct the following model:

$$FEE_{i,t} = a_0 + a_1 EQUITY / DEBT_{i,t} + CONTROLS_{i,t} + \sum FIRM + \sum YEAR + \varepsilon_{i,t} \quad (5)$$

Where *FEE* is the dependent variable measuring audit pricing, *EQUITY* and *DEBT* are the independent variables representing patient capital, *CONTROL* denotes a vector of control variables, and *FIRM* and *YEAR* indicate firm and year fixed effects, respectively. $\varepsilon_{i,t}$ is the random disturbance term. Our primary interest lies in the coefficient a_1 on patient capital. If our hypothesis holds—that patient capital reduces audit pricing—we expect a_1 to be significantly negative.

3.4 Mathematical Definition of SHAP Values

To capture the nonlinear marginal effects of patient capital on audit pricing, we introduce the SHAP (SHapley Additive exPlanations) method (Lundberg and Lee, 2017) in Section 6.4. SHAP values originate from the Shapley value in cooperative game theory and quantify each feature's marginal contribution to the model's predictions. Let the trained LightGBM model be denoted as $f(x)$, where $x = (x_1, x_2, \ldots, x_M)$ is the input vector of M features. The SHAP value of feature i is defined as:

$$\varphi_i = \sum\nolimits_{-}\left\{S \subseteq F \setminus \{i\}\right\}\left[|S|!\left(|F|-|S|-1\right)!/|F|!\right]\cdot\left[f\left(S \cup \{i\}\right) - f\left(S\right)\right] \quad (6)$$

Where F is the set of all features, S is any subset of features excluding i, and $f(S)$ denotes the expected prediction of the model when only the features in subset S are used (with marginal expectations taken over the other features). This definition satisfies three axioms: local accuracy, missingness, and consistency, ensuring that feature contributions are additively decomposable at the sample level, i.e.:

$$f(x) = \varphi_0 + \sum\nolimits_{-}\{i = 1\} \wedge M\varphi_i \tag{7}$$

Where φ_0 is the baseline prediction when all features are set to their mean values. By computing φ_i for each sample, we can characterize the direction and magnitude of the marginal effect of that feature on audit pricing at the individual level, and further reveal the dynamic trajectory of this effect as the feature value varies continuously via dependence plots. Our LightGBM model is trained to predict audit pricing (*FEE*), with the feature set including core variables such as *EQUIT*, *DEBT*, *ROA*, and *AUDOPIN*. Hyperparameters are tuned via 5-fold cross-validation (learning rate = 0.05, tree depth = 6, max leaves = 31), and SHAP values are computed on the training set and then applied to the full sample.

4 Empirical Results and Analysis

4.1 Descriptive Statistics

Table 2 reports the descriptive statistics for the main variables. Among the patient capital measures, strategic equity (EQUITY) has a minimum value of 0.002 and a maximum of 4.281, indicating substantial variation across firms. However, its mean value is 0.285, suggesting that A-share listed companies' shares remain predominantly held by non-institutional and short-term institutional investors, implying that China still needs to actively cultivate patient capital. The standard deviation of relational debt (DEBT) is 0.172, indicating that the proportion of long-term debt does not vary considerably across firms. The descriptive statistics for other variables are all within reasonable ranges. The maximum Variance Inflation Factor (VIF) among the control variables is 1.56, suggesting that multicollinearity is not a serious concern.

Table 2. Descriptive statistics of main variables

Variable	Obs	Mean	Std.	Min	Max
FEE	24219	14.008	0.697	11.918	19.257
EQUITY	24219	0.285	0.515	0.002	4.281
DEBT	24219	0.168	0.172	0	0.717
BRISK	24219	2.918	3.156	0.16	18.071
ARISK	24219	0.222	0.415	0	1
SIZE	24219	22.614	1.27	19.478	26.452
LEV	24219	0.469	0.194	0.032	0.934
ROA	24219	0.03	0.065	-0.416	0.255
GROWTH	24219	0.134	0.395	-0.673	4.474
BOARD	24219	2.304	0.253	1.609	2.996
INDEP	24219	0.382	0.075	0.231	0.615
DUAL	24219	0.24	0.427	0	1
TOP1	24219	0.327	0.146	0.075	0.758
AGE	24219	2.522	0.527	1.609	3.466
AUDOPIN	24219	0.968	0.176	0	1

4.2 Baseline Regression Results

We employ Model (4) to examine the impact of patient capital on audit pricing, with empirical results reported in columns (1) and (2) of Table 3. The coefficients of both strategic equity and relational debt are significantly negative at the 1% level, indicating that the introduction of patient capital into firms reduces audit pricing, thus preliminarily validating Hypothesis H1. The baseline regressions reveal the average treatment effect (ATE)

of patient capital on audit pricing. However, these results leave two deeper questions unanswered: (1) Do the marginal effects of the two types of patient capital exhibit nonlinear evolution as their respective scales vary? (2) Does this nonlinearity provide a quantitative basis for auditors' differentiated pricing decisions? To address these questions, we introduce the SHAP methodology following the heterogeneity analysis, re-examining the risk-transmission logic of the two capital types from a continuous perspective.

Table 3. Baseline regression and robustness test results

	(1)	(2)	(3)	(4)	(5)
	FEE	FEE	Excluding certain samples	Excluding certain samples	Replacing independent variable
EQUITY	-0.0102***		-0.0103***		
	(-2.6753)		(-2.6907)		
DEBT		-0.1032***		-0.0102***	
		(-4.3820)		(-2.6753)	
SUM					-0.1032***
					(-4.3820)
SIZE	0.3457***	0.3517***	0.3457***	0.3457***	0.3517***
	(26.7183)	(27.0681)	(26.7189)	(26.7183)	(27.0681)
LEV	-0.0086	0.0130	-0.0086	-0.0086	0.0130
	(-0.2381)	(0.3512)	(-0.2374)	(-0.2381)	(0.3512)
ROA	-0.3914***	-0.3923***	-0.3914***	-0.3914***	-0.3923***
	(-7.8581)	(-7.8935)	(-7.8582)	(-7.8581)	(-7.8935)
GROWTH	-0.0057	-0.0071	-0.0057	-0.0057	-0.0071
	(-1.0051)	(-1.2551)	(-1.0050)	(-1.0051)	(-1.2551)
BOARD	0.0475***	0.0473***	0.0475***	0.0475***	0.0473***
	(4.4559)	(4.4445)	(4.4558)	(4.4559)	(4.4445)
INDEP	-0.1236***	-0.1236***	-0.1236***	-0.1236***	-0.1236***
	(-3.7445)	(-3.7570)	(-3.7446)	(-3.7445)	(-3.7570)
DUAL	0.0064	0.0063	0.0064	0.0064	0.0063
	(0.7698)	(0.7598)	(0.7698)	(0.7698)	(0.7598)
TOP1	-0.0142	-0.0181	-0.0142	-0.0142	-0.0181
	(-0.2009)	(-0.2556)	(-0.2003)	(-0.2009)	(-0.2556)
AGE	0.2478***	0.2499***	0.2478***	0.2478***	0.2499***
	(8.3083)	(8.4013)	(8.3080)	(8.3083)	(8.4013)
AUDOPIN	-0.0881***	-0.0856***	-0.0881***	-0.0881***	-0.0856***
	(-5.8940)	(-5.7665)	(-5.8938)	(-5.8940)	(-5.7665)
_cons	5.6071***	5.4694***	5.6071***	5.6071***	5.4694***
	(19.3661)	(18.7986)	(19.3666)	(19.3661)	(18.7986)
Firm/Year FE	Yes	Yes	Yes	Yes	Yes
N	23936	23936	23936	23936	23936
R2	0.920	0.920	0.920	0.920	0.920

4.3 Robustness Tests

Excluding Certain Samples. In view of the stock market volatility in 2015 and the impact of the COVID-19 pandemic from 2020 to 2022, we delete the samples from these years and re-estimate Model (4). As shown in columns (3) and (4) of Table 3, the finding that patient capital reduces audit pricing remains significant at the 1% level.

Replacing the Independent Variable. Following prior studies (Zheng et al., 2025), we sum the two ratios of strategic equity and relational debt to construct a composite measure of patient capital (SUM), which is then divided by 100 to adjust the scale. Column (5) of Table 3 shows that the research conclusion remains robust after replacing the independent variable.

Lagging the Independent Variable by One Period. To address potential endogeneity arising from reverse

causality between patient capital and audit pricing, we lag both measures of patient capital by one period. As shown in columns (1) and (2) of Table 4, both strategic equity and relational debt significantly reduce audit pricing, further validating the reliability of our conclusions.

Instrumental Variable Approach. Institutional investors providing patient capital may select well-governed firms for investment to reduce their monitoring costs (Zhen et al., 2021), and such firms tend to have lower audit pricing than comparable firms with poorer governance. To mitigate sample selection bias and reverse causality concerns, we employ a two-stage least squares (2SLS) approach. We use the industry-year mean of strategic institutional investors and the one-period lag of relational debt as instrumental variables (Qiang et al., 2025; Yao et al., 2025), denoted as mean-EQUITY and L.DEBT, respectively. Columns (3) and (4) of Table 4 report the 2SLS results for equity-type patient capital, with a first-stage F-statistic of 17.84. Columns (5) and (6) present the 2SLS results for debt-type patient capital, with a first-stage F-statistic of 179.713. Both F-statistics exceed the threshold for weak instrument tests. The coefficients of both instruments in the second stage are significantly negative, indicating that our conclusions remain valid after accounting for potential endogeneity.

Table 4. Robustness tests

	Independent variable lagged one period	Independent variable lagged one period	First stage	Second stage	First stage	Second stage
mean-EQUITY			0.0077***			
			(9.2945)			
EQUITY				-0.0708**		
				(-2.4168)		
L.EQUITY	-0.0093**					
	(-2.2857)					
L.DEBT		-0.0511**			0.4189***	
		(-2.1819)			(33.7656)	
DEBT						-0.1221***
						(-2.9557)
SIZE	0.3453***	0.3481***	-0.0738***	0.3414***	0.0343***	0.3523***
	(24.5358)	(24.6076)	(-6.0892)	(73.0558)	(10.4995)	(41.1858)
LEV	-0.0458	-0.0380	0.0842*	-0.0034	0.1461***	-0.0202
	(-1.1654)	(-0.9576)	(1.7827)	(-0.1930)	(11.0063)	(-0.7552)
ROA	-0.3989***	-0.4033***	-0.0376	-0.3945***	0.0259	-0.4001***
	(-7.4454)	(-7.5397)	(-0.5827)	(-11.8608)	(1.4418)	(-9.2345)
GROWTH	-0.0056	-0.0063	0.0352***	-0.0036	-0.0081***	-0.0073
	(-0.8658)	(-0.9704)	(4.3130)	(-0.8527)	(-3.4761)	(-1.1443)
BOARD	0.0479***	0.0477***	-0.0147	0.0467***	0.0000	0.0477***
	(4.2341)	(4.2137)	(-0.9082)	(5.6918)	(0.0081)	(4.9239)
INDEP	-0.1263***	-0.1273***	-0.0112	-0.1247***	-0.0010	-0.1275***
	(-3.5852)	(-3.6164)	(-0.2073)	(-4.9749)	(-0.0820)	(-4.3830)
DUAL	0.0073	0.0071	-0.0081	0.0059	-0.0021	0.0068
	(0.8209)	(0.8010)	(-0.7144)	(1.1190)	(-0.7319)	(1.0803)
TOP1	0.0363	0.0313	0.6107***	0.0256	0.0238	0.0343
	(0.4616)	(0.3988)	(6.9642)	(0.7654)	(1.1849)	(0.8005)
AGE	0.3803***	0.3831***	-0.1176***	0.2372***	-0.0206*	0.3806***
	(9.8126)	(9.9000)	(-2.7413)	(15.0203)	(-1.7482)	(16.3828)
AUDOPIN	-0.0798***	-0.0796***	0.0235	-0.0867***	0.0249***	-0.0766***
	(-5.3206)	(-5.3167)	(1.1737)	(-8.7757)	(4.4575)	(-6.2619)
_cons	5.2571***	5.1905***	1.8077***		-0.7240***	
	(16.5130)	(16.2187)	(6.1837)		(-9.1418)	
Firm/Year FE	Yes	Yes	Yes	Yes	Yes	Yes
N	19266	19266	23936	23936	19266	19266
R2	19266	19266	0.372	0.610	0.761	0.564

Entropy Balancing Matching. The relationship between firms' receipt of patient capital investment and audit pricing may be subject to endogeneity concerns. Entropy balancing (Hainmueller, 2012) assigns a set of weights

w_i to the control group samples such that the weighted control group exactly matches the treatment group in the first (mean), second (variance), and third (skewness) moments of covariates, thereby minimizing selection bias while preserving the full sample. Let the treatment indicator be $T_i \in [0,1]$ (where $T_i = 1$ denotes firms with a higher level of patient capital, and $T_i = 0$ otherwise), and let the covariate vector be X_i. Entropy balancing obtains the optimal weights $\{\omega_i\}_\{i: T_i = 0\}$ by solving the following constrained optimization problem:

$$\begin{aligned} &\min_\{\omega_i\} \sum_\{i: T_i = 0\} \omega_i \cdot \log(\omega_i) \\ &s.t. \sum_\{i: T_i = 0\} \omega_i \cdot X_i = \sum_\{i: T_i = 1\} X_i, \sum_\{i: T_i = 0\} \omega_i = 1 \\ &\omega_i \geq 0 \forall i: T_I = 0 \end{aligned} \tag{8}$$

where the objective function is the entropy measure, and the constraints ensure that the weighted control group exactly matches the treatment group in the first moment (in practice, second and third moments are often additionally constrained). Compared with propensity score matching (PSM), entropy balancing avoids efficiency loss due to sample attrition and does not require iterative nearest-neighbor search, rendering it more robust in terms of covariate balance. We split the sample into high-patient-capital and low-patient-capital groups based on the industry-year median of strategic equity and relational debt, respectively, and perform entropy balancing separately. After matching, the standardized covariate biases are all below 5%, indicating substantial reduction of between-group differences. The results show that the coefficients on the core variables remain significantly negative at the 5% level.

5 Mediation Mechanism Tests

Based on the theoretical analysis above, we argue that the introduction of patient capital not only affects firms' operations and financial conditions but also reduces audit pricing by lowering the business risk and audit risk faced by auditors. To test this potential mechanism, we construct the following models:

$$BRISK_{i,t} = b_0 + b_1 EQUITY / DEBT_{i,t} + CONTROLS_{i,t} + \sum FIRM + \sum YEAR + \varepsilon_{i,t} \tag{9}$$

$$ARISK_{i,t} = c_0 + c_1 EQUITY / DEBIT_{i,t} + CONTROLS_{i,t} + \sum FIRM + \sum YEAR + \varepsilon_{i,t} \tag{10}$$

Where *BRISK* and *ARISK* are the potential mediating variables, representing auditors' business risk and audit risk, respectively. The definitions of other variables are the same as those in Model (9).

To ensure the statistical reliability of the mediation effects, beyond observing the signs and significance of b_1 and c_1 in models (9) and (10), we further employ the Sobel (1982) test and the Bootstrap method (1,000 replications) to jointly test the indirect effects. Let â denote the regression coefficient of patient capital on the mediator (*BRISK* or *ARISK*) (i.e., b_1 in model (9) or (10)), and let $\wedge b$ denote the coefficient of the mediator on audit pricing when both the mediator and patient capital are included in the regression (i.e., α_2 in the extended model $FEE = \alpha_0 + \alpha_1 \cdot P + \alpha_2 \cdot Mediator + Controls$). The Sobel test *Z*-statistic is constructed as follows:

$$Z = (\hat{a} \cdot \wedge b) / sqrt(\wedge b^2 \cdot se_\hat{a}^2 + \hat{a}^2 \cdot se_\wedge b^2) \tag{11}$$

Where $se_\hat{a}$ and $se_\hat{b}$ are the standard errors of $\hat{a}$ and $\hat{b}$, respectively. Under the normality assumption, *Z* follows a standard normal distribution. If $|Z| > 1.96$ ($p < 0.05$), the null hypothesis that the indirect effect equals zero is rejected. To address potential small-sample bias, we also report Bias-corrected Bootstrap 95% confidence intervals; if the interval does not contain zero, the mediation effect is further confirmed. Our calculations show that the indirect effects of both strategic equity and relational debt on audit pricing through business risk and audit risk are significant ($p < 0.01$), and the Bootstrap confidence intervals are entirely within the negative range, corroborating the robustness of the mediation pathways.

First, we test the mediating effect of business risk. Following prior studies (Feng and Wen, 2024), we measure business risk as the standard deviation of return on assets over the past three years, capturing the degree of

firm performance uncertainty. A higher value indicates greater business risk faced by auditors. As shown in columns (1) and (2) of Table 5, the coefficients of both strategic equity and relational debt on business risk are significantly negative at the 1% level, indicating that patient capital investment reduces the business risk faced by auditors—that is, firms' operational risk. Existing studies demonstrate that firms' operational risk is positively associated with audit pricing. Therefore, patient capital reduces audit pricing by lowering auditors' business risk.

Second, we test the mediating effect of audit risk. We measure audit risk using whether the firm has engaged in financial restatement in the current year, coded as 1 if a restatement occurs and 0 otherwise. Firms with financial restatements face greater audit risk than those without. As shown in columns (3) and (4) of Table 5, the coefficients of both strategic equity and relational debt on audit risk are significantly negative, indicating that patient capital reduces audit risk. Lower risk premiums are associated with lower audit pricing.

Table 5. Descriptive statistics of main variables

	BRISK	BRISK	ARISK	ARISK
EQUITY	-0.1877***		-0.0269***	
	(-5.0591)		(-5.6284)	
DEBT		-0.4848**		-0.0496**
		(-2.0339)		(-2.0684)
SIZE	-0.8386***	-0.8004***	-0.0026	0.0019
	(-9.2916)	(-8.7530)	(-0.3630)	(0.2660)
LEV	0.5508	0.6404*	0.0420	0.0505
	(1.4937)	(1.7140)	(1.3894)	(1.6344)
ROA	-19.1994***	-19.1964***	-0.0000	0.0007
	(-20.4486)	(-20.4503)	(-0.0006)	(0.0116)
GROWTH	0.3294***	0.3178***	0.0115	0.0100
	(4.9825)	(4.8074)	(1.4910)	(1.2991)
BOARD	0.4677***	0.4689***	0.0267*	0.0269*
	(4.0507)	(4.0533)	(1.9097)	(1.9237)
INDEP	-0.1042	-0.1018	0.0751*	0.0755*
	(-0.3147)	(-0.3074)	(1.7534)	(1.7604)
DUAL	0.1316	0.1324	0.0119	0.0121
	(1.3699)	(1.3740)	(1.2685)	(1.2857)
TOP1	-1.1757**	-1.2856***	0.0082	-0.0081
	(-2.4467)	(-2.6715)	(0.1709)	(-0.1682)
AGE	1.9718***	2.0062***	-0.0124	-0.0075
	(7.7231)	(7.8566)	(-0.4991)	(-0.3020)
AUDOPIN	-1.4109***	-1.4025***	-0.0267	-0.0260
	(-5.7836)	(-5.7489)	(-1.5769)	(-1.5303)
_cons	17.9557***	17.0160***	0.2293	0.1171
	(8.8642)	(8.3183)	(1.3928)	(0.7013)
Firm/Year FE	Yes	Yes	Yes	Yes
N	23936	23936	23936	23936
R2	0.546	0.545	0.371	0.371

6 Heterogeneity Analysis

6.1 Ownership Concentration

Ownership concentration is a crucial determinant of corporate governance structure, significantly influencing internal monitoring and risk prevention. Consequently, the impact of patient capital on audit pricing may vary with ownership concentration. We measure ownership concentration using the shareholding ratio of the largest shareholder and examine the effect of patient capital on audit pricing separately for firms with ownership concentration above and below the sample mean. As shown in Table 6, in firms with low ownership concentration, both strategic equity and relational debt significantly reduce audit pricing, whereas the effect is insignificant in firms with high ownership concentration. The likely explanation is that firms with low ownership

concentration face a more pronounced "free-rider" problem, where shareholders lack sufficient incentives and capabilities to closely monitor management. In the absence of effective monitoring, information asymmetry intensifies, increasing operational risk and the likelihood of financial fraud. However, when patient capital is introduced, institutional investors step in as "active monitors," improving corporate governance, enhancing internal controls, optimizing the information environment, and reducing auditors' information acquisition costs, thereby significantly lowering auditors' business risk and audit risk, and consequently reducing audit pricing.

Table 6. Heterogeneity results of ownership concentration on the impact of patient capital on audit pricing

	Low ownership concentration	High ownership concentration	Low ownership concentration	High ownership concentration
EQUITY	-0.0175**	-0.0048		
	(-2.4995)	(-1.1301)		
DEBT			-0.1131***	-0.0515
			(-3.4905)	(-1.5754)
SIZE	0.3086***	0.3837***	0.3155***	0.3866***
	(19.0771)	(16.2700)	(19.5053)	(16.2872)
LEV	0.0533	-0.1888***	0.0790*	-0.1777***
	(1.2150)	(-3.5935)	(1.7797)	(-3.3284)
ROA	-0.2998***	-0.4290***	-0.3000***	-0.4304***
	(-5.5460)	(-5.2652)	(-5.5598)	(-5.2797)
GROWTH	-0.0056	-0.0019	-0.0067	-0.0030
	(-0.6691)	(-0.2611)	(-0.8007)	(-0.4009)
BOARD	0.0485***	0.0406**	0.0475***	0.0412***
	(3.6718)	(2.5469)	(3.6045)	(2.5933)
INDEP	-0.1115***	-0.1392***	-0.1104***	-0.1395***
	(-2.7083)	(-3.0463)	(-2.6926)	(-3.0561)
DUAL	-0.0003	0.0173	-0.0009	0.0178
	(-0.0311)	(1.2269)	(-0.0880)	(1.2645)
TOP1	-0.3764***	0.1931*	-0.3756***	0.1890*
	(-2.6900)	(1.8668)	(-2.6991)	(1.8309)
AGE	0.2347***	0.1972***	0.2380***	0.1978***
	(5.5778)	(4.1751)	(5.6659)	(4.1977)
AUDOPIN	-0.0710***	-0.0490**	-0.0686***	-0.0471**
	(-4.1179)	(-2.1590)	(-4.0298)	(-2.0584)
_cons	6.5219***	4.8067***	6.3600***	4.7384***
	(18.7411)	(8.6148)	(18.2541)	(8.4420)
Firm FE	yes	yes	yes	yes
Year FE	yes	yes	yes	yes
N	13068	10386	13068	10386
R2	0.918	0.942	0.918	0.942

6.2 Degree of Market Competition

Under different market competition environments, firms' development strategies and behavioral choices vary, which may also lead to differential effects of patient capital on audit pricing. We measure the "degree of market competition" using the Herfindahl-Hirschman Index (HHI), where a higher value indicates lower competition in the firm's industry. As shown in Table 7, when firms operate in highly competitive industries, both strategic equity and relational debt significantly reduce audit pricing. The likely explanation is that intense market competition compresses firms' profit margins, intensifies market rivalry, and increases firms' market risk. Following the infusion of patient capital, on the one hand, investors participate more actively in corporate governance, monitoring managerial decisions to prevent myopic behavior and financial fraud driven by short-term profit motives, thereby ensuring the maximization of firms' long-term interests. On the other hand, investors broadly absorb external environmental factors—such as industry trends, innovation frontiers, and competitive dynamics (Hu et al., 2025)—to assist firms in strategic decision-making and improve operating conditions through efficient financial resource allocation. These features, which reduce firms' operational risk and the likelihood of financial fraud, effectively mitigate auditors' business risk and audit risk, thereby lowering audit pricing.

Table 7. Heterogeneity results of market competition on the impact of patient capital on audit pricing

	High competition	Low competition	High competition	Low competition
EQUITY	-0.0139**	-0.0040		
	(-2.1374)	(-0.8802)		
DEBT			-0.0927***	-0.0717*
			(-3.1399)	(-1.9571)
SIZE	0.3051***	0.4009***	0.3109***	0.4044***
	(19.7097)	(15.4928)	(20.0259)	(15.5593)
LEV	0.0428	-0.2124***	0.0633	-0.1962***
	(0.9785)	(-3.6544)	(1.4341)	(-3.3118)
ROA	-0.3103***	-0.3852***	-0.3117***	-0.3881***
	(-5.9952)	(-4.1441)	(-6.0351)	(-4.1753)
GROWTH	-0.0059	-0.0112	-0.0070	-0.0123
	(-0.7226)	(-1.5000)	(-0.8600)	(-1.6313)
BOARD	0.0482***	0.0480***	0.0476***	0.0487***
	(3.8351)	(2.7872)	(3.7925)	(2.8374)
INDEP	-0.1214***	-0.1458***	-0.1214***	-0.1460***
	(-3.1888)	(-2.9269)	(-3.2013)	(-2.9366)
DUAL	0.0015	0.0184	0.0011	0.0188
	(0.1571)	(1.1011)	(0.1144)	(1.1285)
TOP1	-0.3420***	0.0891	-0.3432***	0.0888
	(-2.7351)	(0.7412)	(-2.7493)	(0.7428)
AGE	0.2275***	0.2227***	0.2303***	0.2230***
	(5.7875)	(4.4803)	(5.8717)	(4.5055)
AUDOPIN	-0.0670***	-0.0763***	-0.0645***	-0.0744***
	(-4.0646)	(-3.1719)	(-3.9554)	(-3.0738)
_cons	6.6035***	4.4284***	6.4673***	4.3468***
	(19.7972)	(7.1101)	(19.3244)	(6.9516)
Firm FE	yes	yes	yes	yes
Year FE	yes	yes	yes	yes
N	14319	9099	14319	9099
R2	0.915	0.943	0.916	0.943
	High competition	Low competition	High competition	Low competition

6.3 Digital Transformation

Digital transformation enhances firms' production efficiency and information transparency, while also introducing varying degrees of inherent risk and control risk. Consequently, whether a firm undergoes digital transformation may affect the role of patient capital in audit pricing. We measure digital transformation using a dummy variable, coded as 1 if the firm has undergone digital transformation and 0 otherwise. As shown in Table 8, for firms undergoing digital transformation, the introduction of patient capital significantly reduces audit pricing, whereas the effect is not significant for firms that have not undergone digital transformation. The likely explanation is that, compared with non-transforming firms, digitalizing firms face high inherent risks such as business model transformation, as well as control risks including legacy system migration and data security. Auditors need to invest more resources in IT auditing and data analytics, perform more complex substantive procedures, increase audit costs, and charge higher risk premiums. Meanwhile, digital transformation is typically not achieved overnight but entails sustained long-term capital and talent investments, exposing firms to greater operational risk, which in turn feeds back into higher audit pricing. However, the infusion of patient capital strengthens governance and oversight, ensuring that firms establish robust governance frameworks, risk assessment systems, and internal control mechanisms for digital investments, thereby systematically reducing inherent and control risks. At the same time, patient capital understands the long-term nature of digital transformation, promotes it through financial support, reduces information asymmetry between investors and firms, and encourages long-term investment. For auditors, recognizing a firm's patient capital investment signals more reasonable transformation motives and a more stable financial environment, correspondingly lowering the risk of material misstatement at the financial statement level. Through IT auditing, auditors gain greater reliance on internal controls, reducing the need for extensive substantive testing, thereby lowering audit input and audit risk, and consequently reducing audit pricing.

Table 8. Heterogeneous results of digital transformation of patient capital affecting audit pricing

	(1)	(2)	(3)	(4)
	Digital transformation	Not undergoing digital transformation	Digital transformation	Not undergoing digital transformation
EQUITY	-0.0106***	-0.0114		
	(-2.6875)	(-0.8669)		
DEBT			-0.1094***	-0.0668
			(-4.6382)	(-0.4326)
SIZE	0.3469***	0.2012***	0.3533***	0.2074***
	(26.5900)	(3.6889)	(26.9695)	(3.8479)
LEV	-0.0075	-0.1953	0.0161	-0.1809
	(-0.2054)	(-1.0170)	(0.4349)	(-0.9182)
ROA	-0.3674***	0.1462	-0.3682***	0.1560
	(-7.5629)	(0.3550)	(-7.5912)	(0.3802)
GROWTH	-0.0072	0.0016	-0.0088	0.0009
	(-1.2144)	(0.0595)	(-1.4776)	(0.0348)
BOARD	0.0479***	0.0756	0.0479***	0.0746
	(4.4786)	(1.2284)	(4.4873)	(1.2065)
INDEP	-0.1348***	0.1077	-0.1347***	0.1034
	(-4.1713)	(0.5857)	(-4.1801)	(0.5601)
DUAL	0.0065	0.0201	0.0065	0.0220
	(0.7657)	(0.4562)	(0.7634)	(0.4985)
TOP1	-0.0097	0.5467*	-0.0133	0.5483*
	(-0.1371)	(1.7568)	(-0.1889)	(1.7392)
AGE	0.2552***	0.1707	0.2578***	0.1471
	(8.4936)	(0.7657)	(8.6131)	(0.6902)
AUDOPIN	-0.0892***	0.0187	-0.0862***	0.0164
	(-5.8627)	(0.3158)	(-5.7169)	(0.2686)
_cons	5.5676***	8.3024***	5.4183***	8.2334***
	(18.9987)	(6.0972)	(18.4108)	(5.9653)
Firm FE	yes	yes	yes	yes
Year FE	yes	yes	yes	yes
N	23211	315	23211	315
R^2	0.921	0.957	0.921	0.957

Synthesizing the above heterogeneity results, the three pathways converge on a consistent theoretical logic: the magnitude of patient capital's effect on audit pricing depends on "signaling efficiency." When firms face severe information asymmetry (dispersed ownership), greater external pressure (intense market competition), or internal governance restructuring (digital transformation), the risk-mitigation signal of patient capital becomes scarcer and more valuable, yielding stronger marginal effects on audit pricing. This "signaling efficiency" framework provides a unified theoretical explanation for the boundary conditions of patient capital's role, while also establishing the logical foundation for the differentiated patterns revealed in the subsequent SHAP nonlinear analysis.

6.4 SHAP-Based Nonlinear Heterogeneity Analysis

The subgroup regression analyses in the preceding sections (Table 6 to Table 8) examine the moderating effects of ownership concentration, market competition, and digital transformation through mean-based dichotomization. However, this approach loses continuous individual-level information and fails to reveal the dynamic trajectory of core effects as moderating variables vary continuously. To address this limitation, we introduce the SHAP (SHapley Additive exPlanations) method. Based on a LightGBM model (trained to predict audit pricing FEE), we compute the marginal contribution (SHAP values) of each feature to the predicted output. Leveraging 24,219 valid observations, we examine the dynamic marginal effects of patient capital on audit pricing from a continuous perspective. This section retains only four variables—EQUITY, DEBT, ROA, and AUDOPIN—and the following conclusions should be regarded as supplementary explorations complementing the main regression results.

Fig. 1 reports the feature importance ranking, where the horizontal axis represents the feature importance values and the vertical axis lists variable names; larger values indicate stronger explanatory power for audit pricing. EQUITY, ROA, and DEBT rank among the top three, indicating that patient capital is a core factor influencing audit fees, while auditor opinion (AUDOPIN) exhibits the lowest contribution, consistent with expectations.

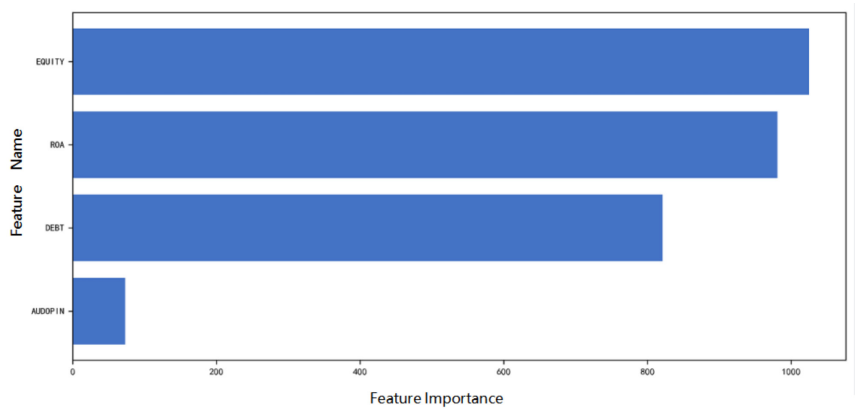


Fig. 1. LightGBM model feature importance ranking

Fig. 2 presents the SHAP bee swarm plot. The vertical axis ranks features in descending order of global importance; the horizontal axis represents SHAP values, where positive values indicate upward pressure on audit pricing and negative values indicate downward pressure; the color represents the value of the variable itself, with red indicating high feature values and blue indicating low feature values. For EQUITY and DEBT, low-value samples (blue) are concentrated in the negative range, while high-value samples (red) are largely distributed in the positive range. This distribution pattern does not contradict the baseline regression finding that "patient capital reduces audit pricing on average"—the baseline regression captures the conditional average effect, while SHAP reveals the marginal upward-pushing effect of high-value samples. These two correspond respectively to the central tendency and extreme tail characteristics of the explanatory variables, complementing each other.

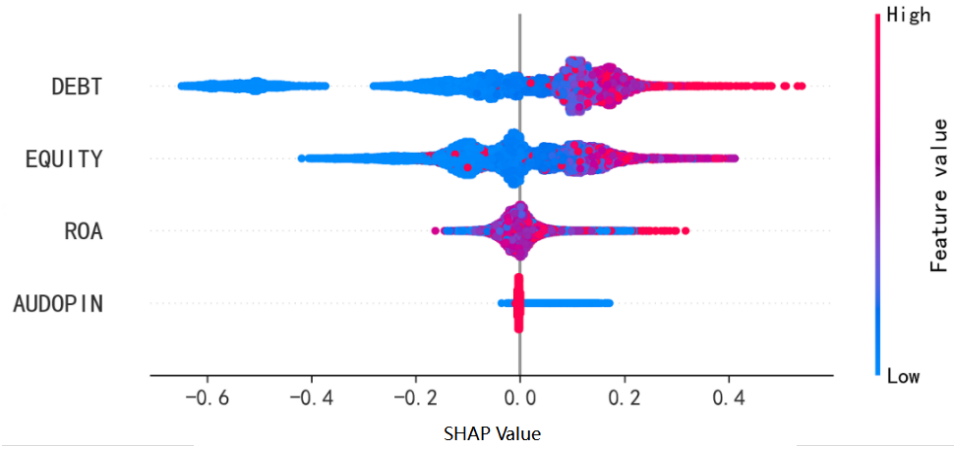


Fig. 2. SHAP bee swarm plot

Fig. 3 presents the SHAP dependence plot for EQUITY. The horizontal axis represents the value of EQUITY itself, and the vertical axis represents the marginal SHAP contribution of EQUITY. The color bar on the right

indicates the level of DEBT, which is used to observe the joint distribution characteristics of the two patient capital variables. The marginal contribution of EQUITY exhibits an asymmetric inverted U-shape: it fluctuates sharply between positive and negative values in the 0–0.2 interval (peaking at 0.25); it turns positive and stabilizes in the 0.2–0.4 interval; and it declines to a slight negative value of approximately −0.05 in the high-equity interval of 0.4–2.0, indicating that high equity levels only maintain limited risk mitigation. The color bar on the right shows that high EQUITY samples are accompanied by declining DEBT, suggesting that the capital structure combination of high equity and low debt is an important reason for the weakening of the effect in this interval. This pattern reveals that auditors regard equity capital as a "safety cushion"—the larger the scale, the more sufficient the risk buffer, but the marginal effect diminishes.

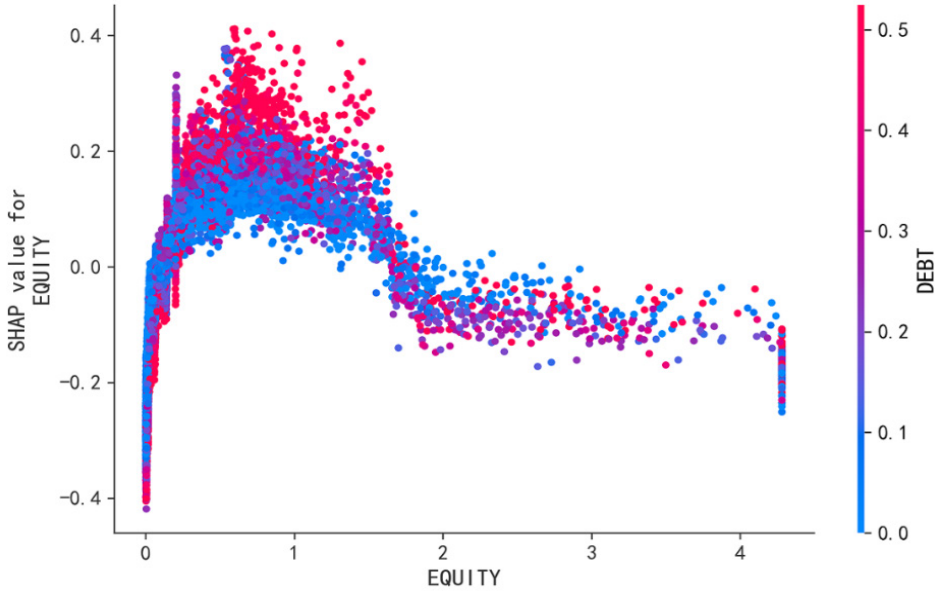


Fig. 3. EQUITY SHAP dependence plot

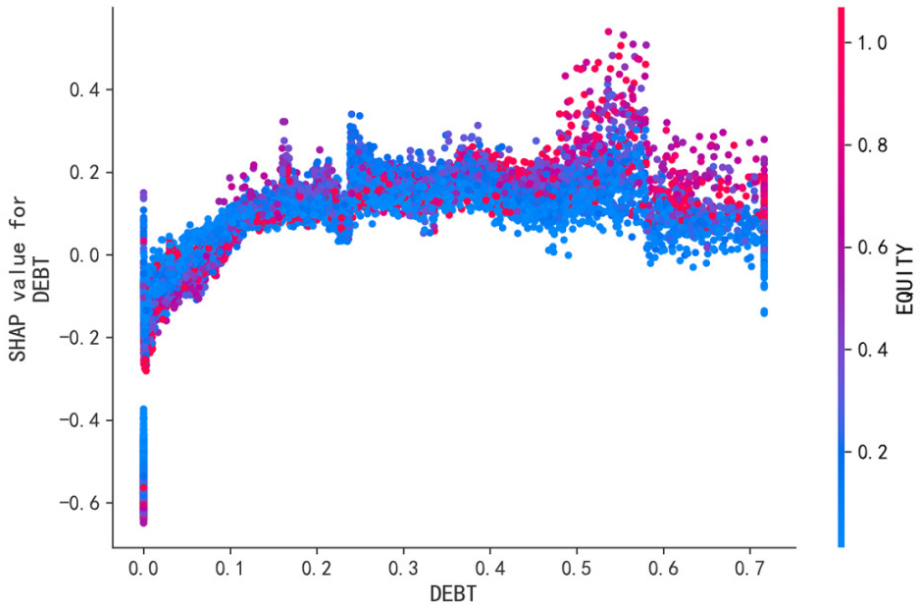


Fig. 4. DEBT SHAP dependence plot

Fig. 4 presents the SHAP dependence plot for DEBT. DEBT exhibits a “negative-to-positive” monotonic inflection-point pattern: in the 0–0.5 interval, the effect is predominantly negative suppression (with governance constraints and tax shield effects dominating); after breaking through 0.5–0.7, it jumps to the positive interval and remains positive thereafter, though the upward slope flattens in the high-debt range (>0.7), suggesting that auditors’ risk premium pricing for excessive debt exhibits diminishing marginal effects. This pattern reveals that debt capital follows a “threshold response” logic—moderate debt serves as a governance tool, while excessive debt becomes a source of risk.

To quantify the nonlinear patterns in the SHAP dependence plots, we conduct quadratic fitting and piecewise linear regression, respectively.

For *EQUITY*, we specify the quadratic model of SHAP values on its own value x:

$$SHAP_EQUITY(x) = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2 + \varepsilon \tag{12}$$

The estimation yields $\beta_2 < 0$ ($p < 0.01$), with the extremum point $x^* = -\beta_1/(2\beta_2) \approx 0.38$, indicating that the marginal mitigation effect of equity capital saturates beyond approximately 38%, corroborating the “safety cushion” logic.

For *DEBT*, we apply piecewise regression with inflection points:

$$\begin{cases} SHAP_DEBT(x) = \alpha_1 \cdot x + \delta_1, x \le \tau_1 \\ \alpha_2 \cdot x + \delta_2, \tau_1 \le x \le \tau_2 \\ \alpha_3 \cdot x + \delta_3, x > \tau_2 \end{cases} \tag{13}$$

The optimal inflection points are determined via grid search as $\tau_1 \approx 0.52$ and $\tau_1 \approx 0.68$, with $\alpha_1 < 0$ ($p < 0.01$), $\alpha_2 > 0$ ($p < 0.05$), and $\alpha_3 > 0$ but substantially smaller than α_2 . These results confirm that debt capital exerts governance effects below 52% leverage, while its risk-enhancing effect decelerates beyond 68%, validating the "threshold response" logic. The “safety cushion” and “threshold response” patterns revealed by the SHAP nonlinear analysis theoretically answer the core question posed in the introduction: auditors do not simply treat all patient capital uniformly; rather, they engage in differentiated risk pricing based on the divergent governance logics of the two capital types. This finding revises the theoretical presumption in extant literature that treats patient capital as a homogeneous construct, and suggests that future research should fully account for the heterogeneous transmission pathways of equity-based versus debt-based patient capital when constructing measurement frameworks for patient capital.

In summary, EQUITY follows a “safety cushion” logic, while DEBT follows a “threshold response” logic, and the marginal transmission mechanisms of the two types of patient capital exhibit fundamental differentiation. This nonlinear evidence provides a quantitative basis for auditors to differentially assess the risk premiums of the two types of capital, and also indicates that traditional linear regression, which simply incorporates both types of capital into a unified “patient” framework, may obscure their heterogeneous transmission paths.

7 Conclusions and Implications

Using a sample of Shanghai and Shenzhen A-share listed companies from 2011 to 2024, this paper empirically examines the impact of patient capital on audit pricing. The findings are as follows: (1) Patient capital significantly reduces audit pricing, with business risk and audit risk serving as the transmission channels; (2) this inhibitory effect is more pronounced in firms with low ownership concentration, intense market competition, and those undergoing digital transformation; and (3) SHAP-based nonlinear analysis reveals a fundamental differentiation in the marginal effects of equity capital and debt capital—EQUITY follows a “safety cushion” logic, with marginal mitigation effects tending to saturate in the high-equity range; DEBT follows a “threshold response” logic, and only when leverage does not exceed the critical point (approximately 0.5–0.7) does it justify a pricing discount.

The implications of this study are threefold. First, it is necessary to strengthen national policy support and guidance to continuously cultivate patient capital. On the one hand, the guiding role of state-owned funds should be fully leveraged, reinforcing the long-term lending support to enterprises through the state-bank-dominated

credit system, promoting institutional investors' use of long-term capital to support sustainable development, and clarifying their guidelines for participating in corporate governance as "active shareholders." On the other hand, in the information disclosure rules for listed companies, enterprises should be encouraged or required to disclose their long-term strategic objectives, communications with shareholders, and information on the construction of risk management mechanisms for long-term value creation, thereby providing the market (auditors) with a basis for identifying firms with "patient capital." Second, enterprises should proactively introduce patient capital, leverage its advantages to elevate risk prevention and control to the core level of corporate governance, and transform it into communicable "credit assets." First, enterprises should strengthen their internal capabilities, attracting patient capital through technological innovation and sound governance. Second, after receiving patient capital, they should seize this opportunity to further improve governance, manage operational and financial risks, and increase innovation investment to enhance innovation quality and resilience, thereby forming a sustainable development loop of "capital empowers enterprises, enterprises return capital." Finally, in communications with auditors and other investors, enterprises should strategically present their clear long-term strategies, robust governance structures, and effective risk prevention systems, so as to strive for more favorable conditions in audit pricing, financing costs, and other aspects. Third, accounting firms should optimize audit risk assessment models and implement differentiated and precise pricing. The "shareholder characteristics" and "whether the firm has patient capital" should be incorporated into considerations. For firms assessed as having mature risk controls, risk premiums may be appropriately reduced. However, it should be noted that the two types of patient capital operate under different logics—equity capital can be regarded as a stable risk-mitigation signal, whereas debt capital only provides a reasonable basis for pricing discounts when the leverage ratio does not exceed the critical threshold (approximately 0.5–0.7). High-debt firms should not be indiscriminately granted low-risk premiums simply because they are labeled as having "patient capital."

This paper has several limitations that point to future research directions. First, constrained by data availability, we are unable to distinguish the differential effects of various types of long-term debt (e.g., bank loans, corporate bonds, convertible bonds) on audit pricing; future research could refine this decomposition using hand-collected debt contract data. Second, this paper focuses on audit pricing as the outcome variable without examining the impact of patient capital on audit inputs (e.g., audit hours, audit procedure complexity); future research could supplement evidence from the perspective of the audit production process. Third, our sample is confined to China's A-share market; whether our findings generalize to other emerging economies or developed markets awaits cross-country comparative validation.

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