# Low-Complexity Parallel Systolic Architectures for Computing Multiplication and Squaring over GF( $2^{m}$ ) 

Chiou-Yng Lee*<br>Department of Computer Information and Network Engineering<br>Lunghwa University of Science and Technology<br>Taoyuan, Taiwan<br>PP010@mail.lhu.edu.tw

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#### Abstract

Recently, cryptographic applications based on finite fields have attracted much interest. This paper presents a unified systolic multiplier under the method of the multiply-by- $x^{2}$ and the folded technique. This circuit is particularly suitable for implementing multiplication and squaring in $\operatorname{GF}\left(2^{m}\right)$. The results show that our proposed multiplier saves up to $75 \%$ space complexity and $50 \%$ latency as compared to the traditional multipliers proposed by Yeh et al. and Wang-Lin. Also, the proposed squarer saves about $45 \%$ space complexity as compared to the traditional squarer presented by Guo and Wang.


Keywords: finite field, polynomial basis, systolic architecture, MSB-first multiplication algorithm

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[^0]:    * Correspondence author

